Kickoff

Dr.-Ing. Kailai Li

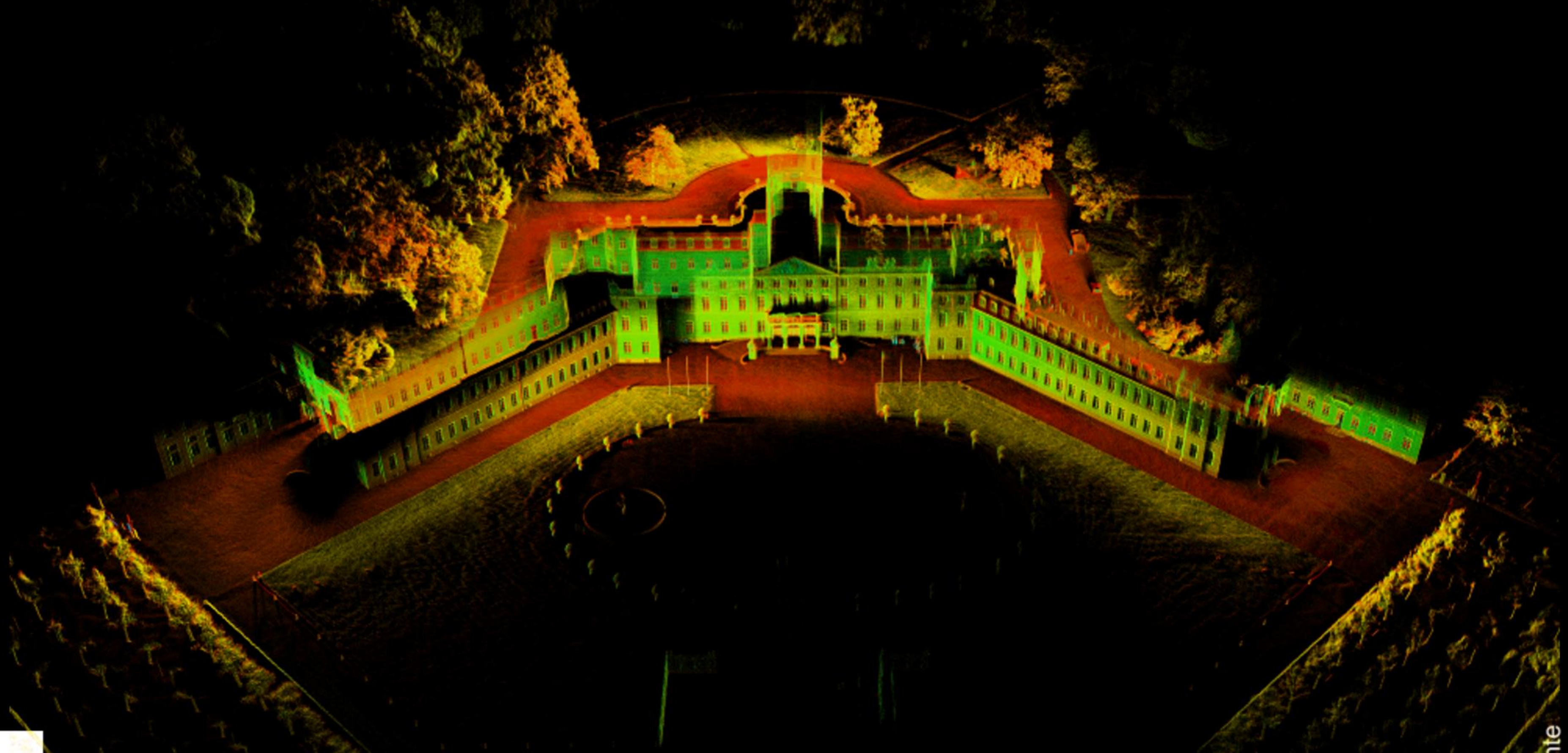
LMA-Exercise 0 | April 25, 2022

About LMA-Exercise

- weekly (almost) at 2pm, 50.34 Room-101
- language: English/German (also for your oral exams)
- duration: 1 - 1.5 h
- content
  1) lecture-related exercises (sheets + answers + notes) → uploaded to ILIAS
  2) research review on state-of-the-art state estimation techniques for autonomous and mobile robots → **new in SS22, held irregularly, exam-irrelevant**

Towards High-Performance Solid-State-LiDAR-Inertial Odometry and Mapping

Kailai Li, Meng Li, and Uwe D. Hanebeck



Karlsruhe Institute of Technology

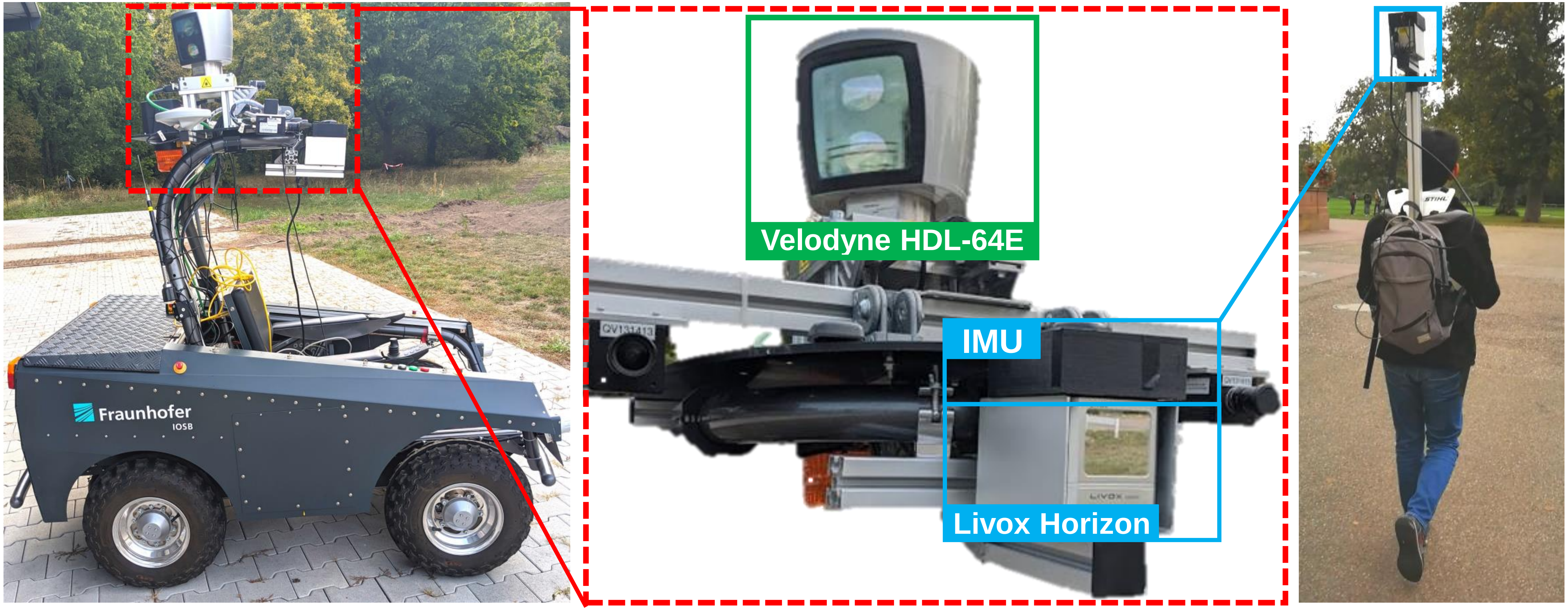
Intelligent Sensor-Actuator-Systems Lab (ISAS), Institute of Anthropomatics and Robotics
Karlsruhe Institute of Technology (KIT)

Intelligente Sensor-Aktor-Systeme

Towards High-Performance Solid-State-LiDAR-Inertial Odometry and Mapping

LiLi-OM (LIvox LiDAR-Inertial Odometry and Mapping)

- cost-effective, real-time LiDAR-inertial odometry and mapping system for onboard setup
- applicable for both solid-state and conventional LiDARs



Reference: Kailai Li, Meng Li, and Uwe D. Hanebeck. Towards High-Performance Solid-State-LiDAR-Inertial Odometry and Mapping. *IEEE Robotics and Automation Letters*, 6(3):5167–5174, 2021.

About the Talk

LiLi-OM (LIvox LiDAR-Inertial Odometry and Mapping)

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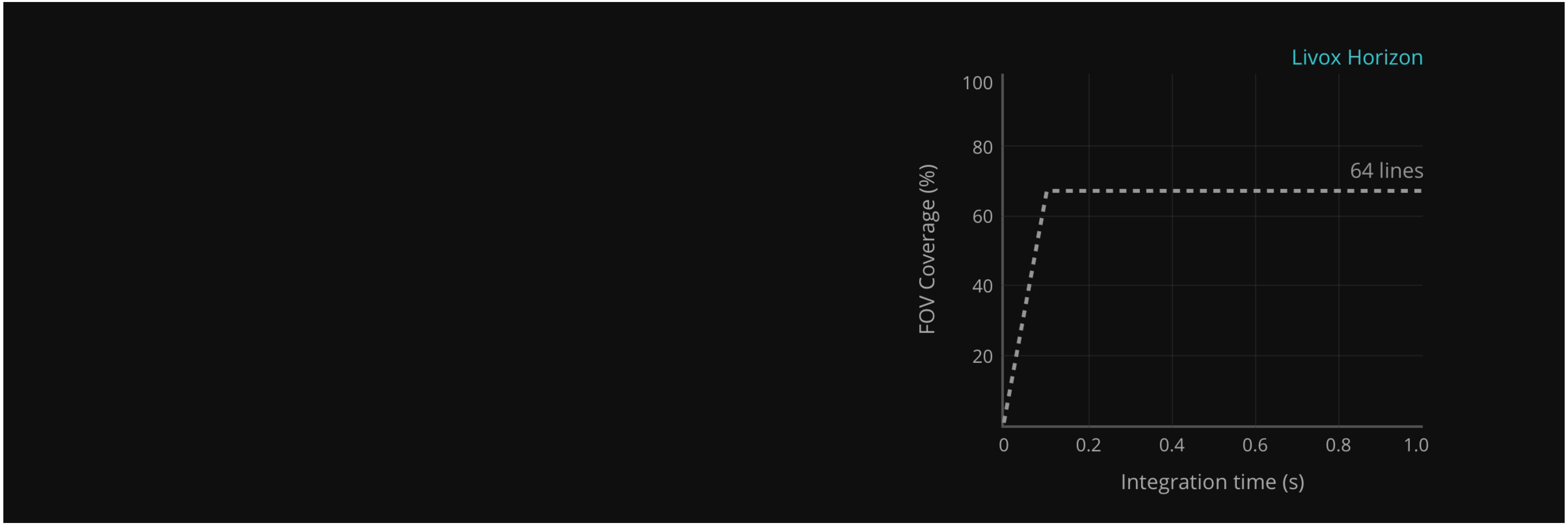
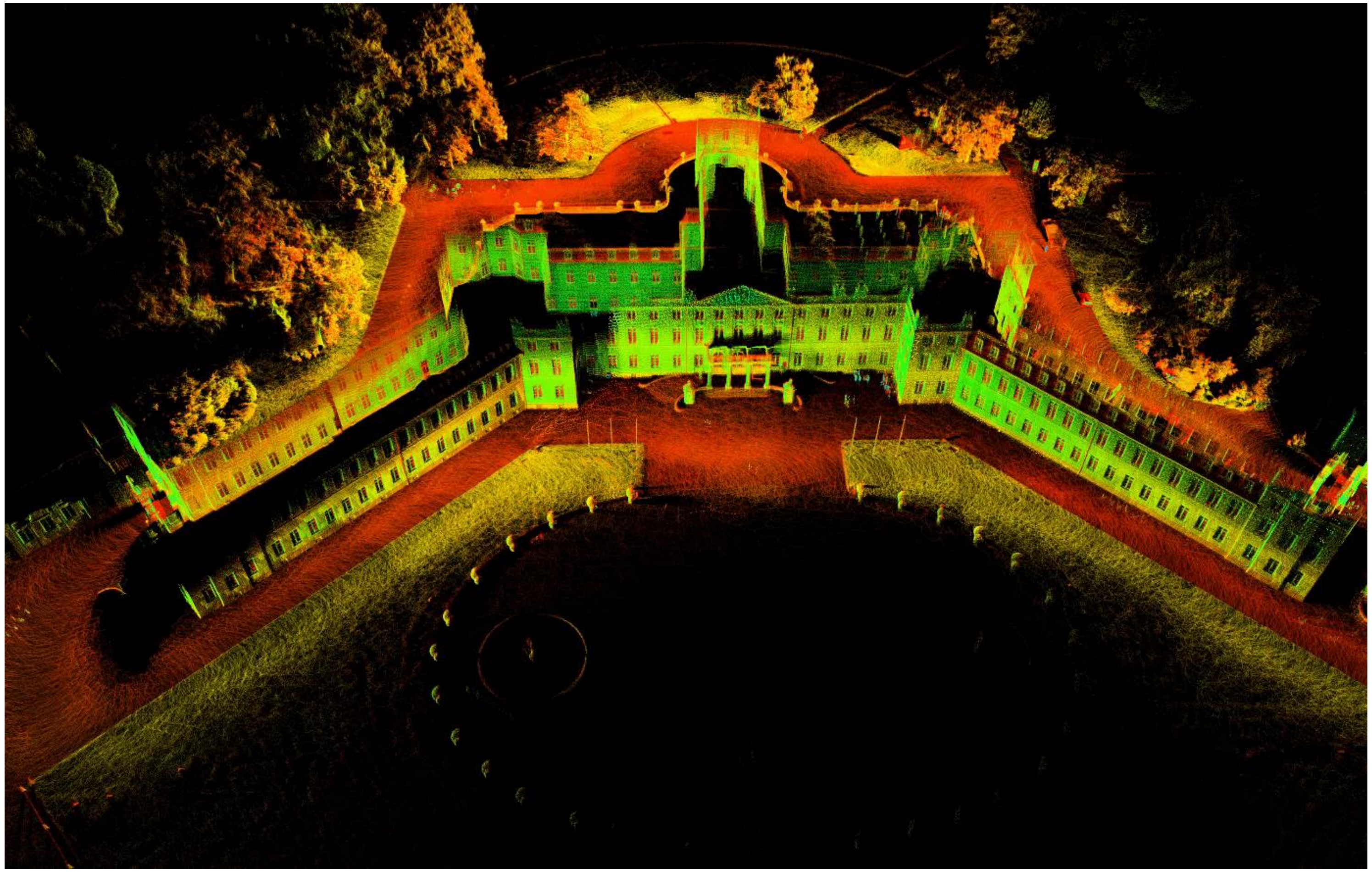


Reference: https://www.livoxtech.com/

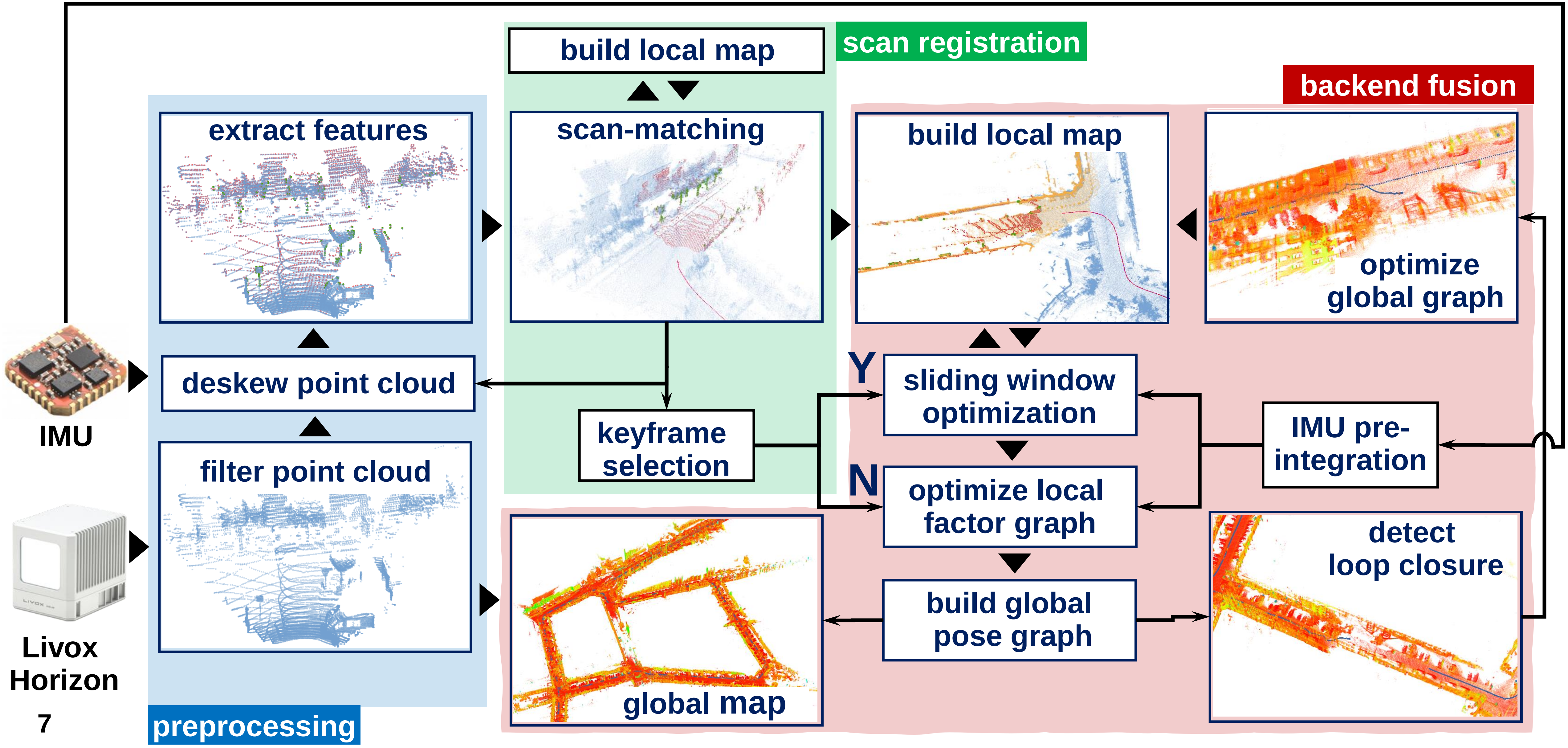
LiDAR-Inertial Odometry and Mapping

Problem formulation

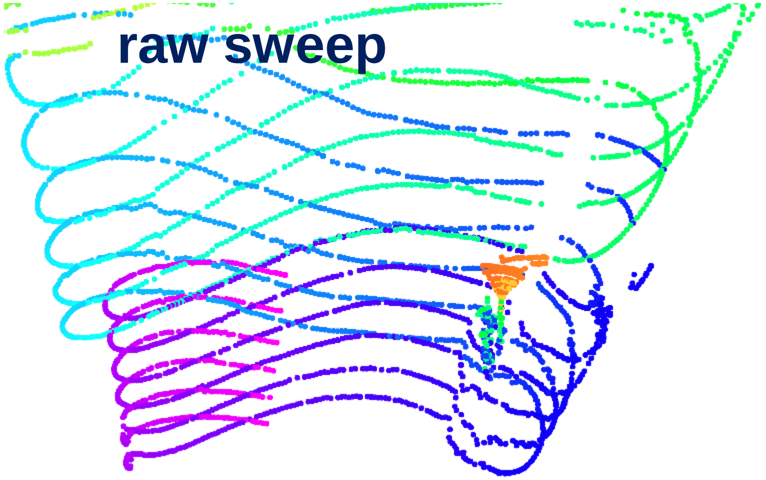
- estimate 6 DoF pose of onboard sensor suite at LiDAR frame rate (10Hz)
- build a 3D map simultaneously

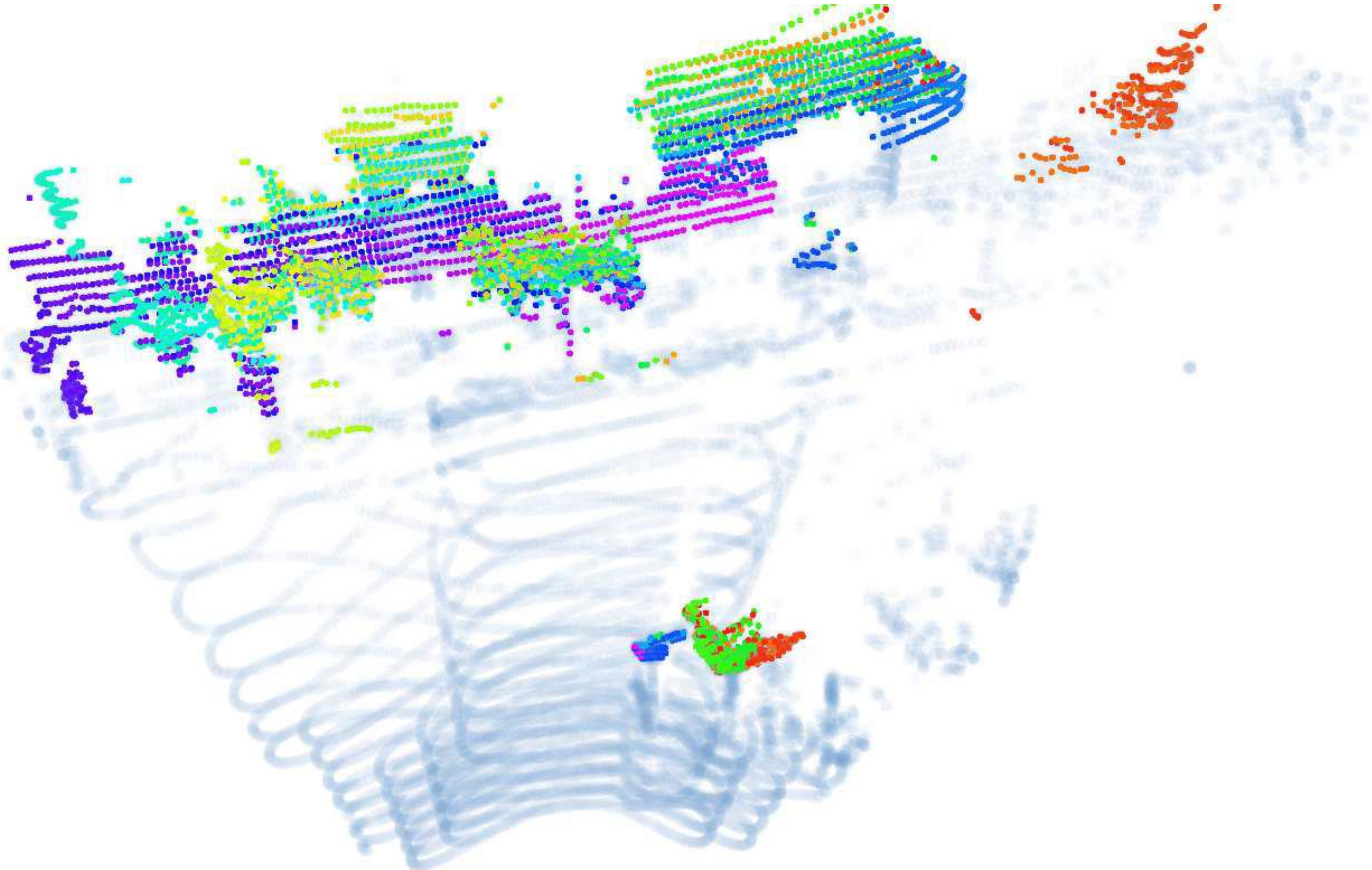


System Pipeline

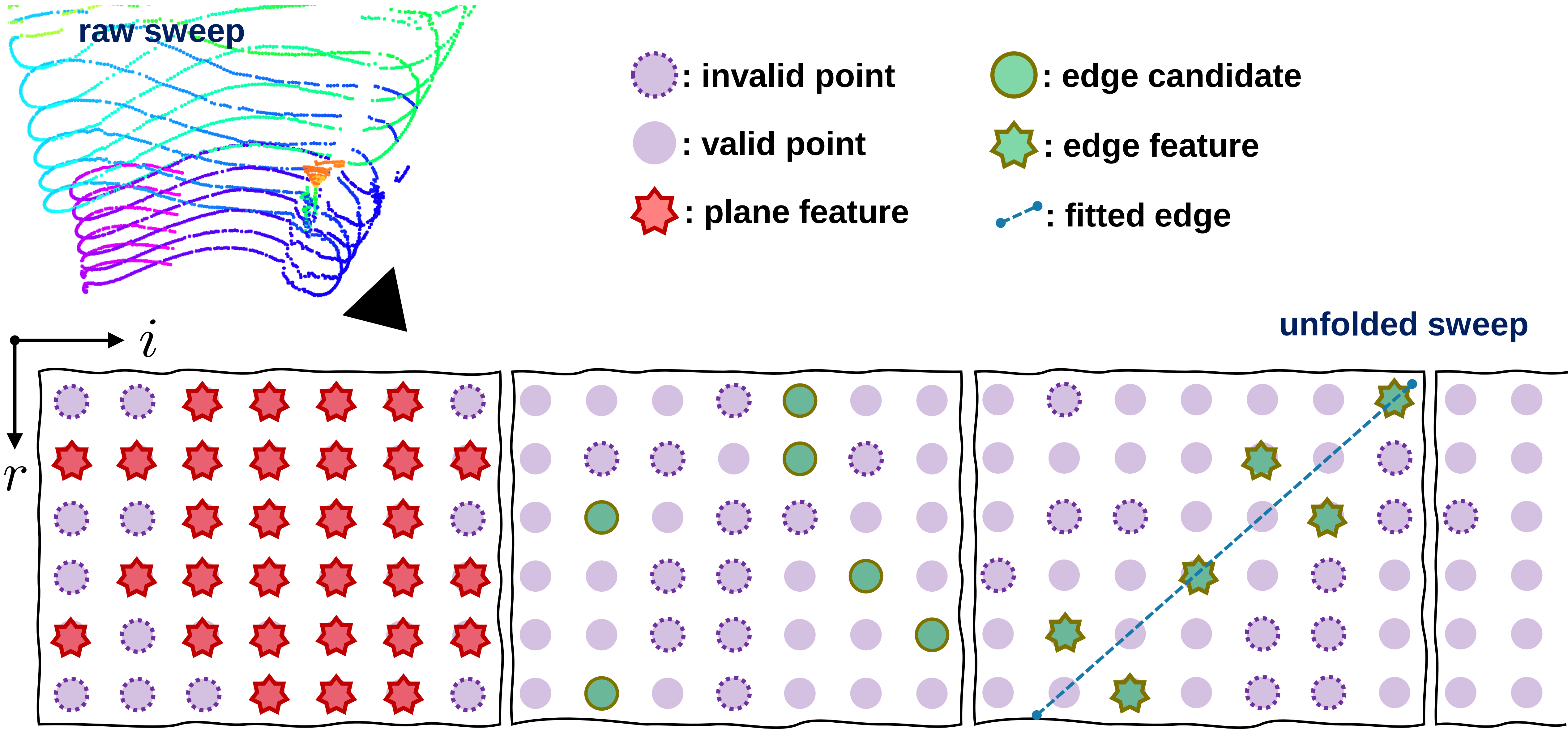


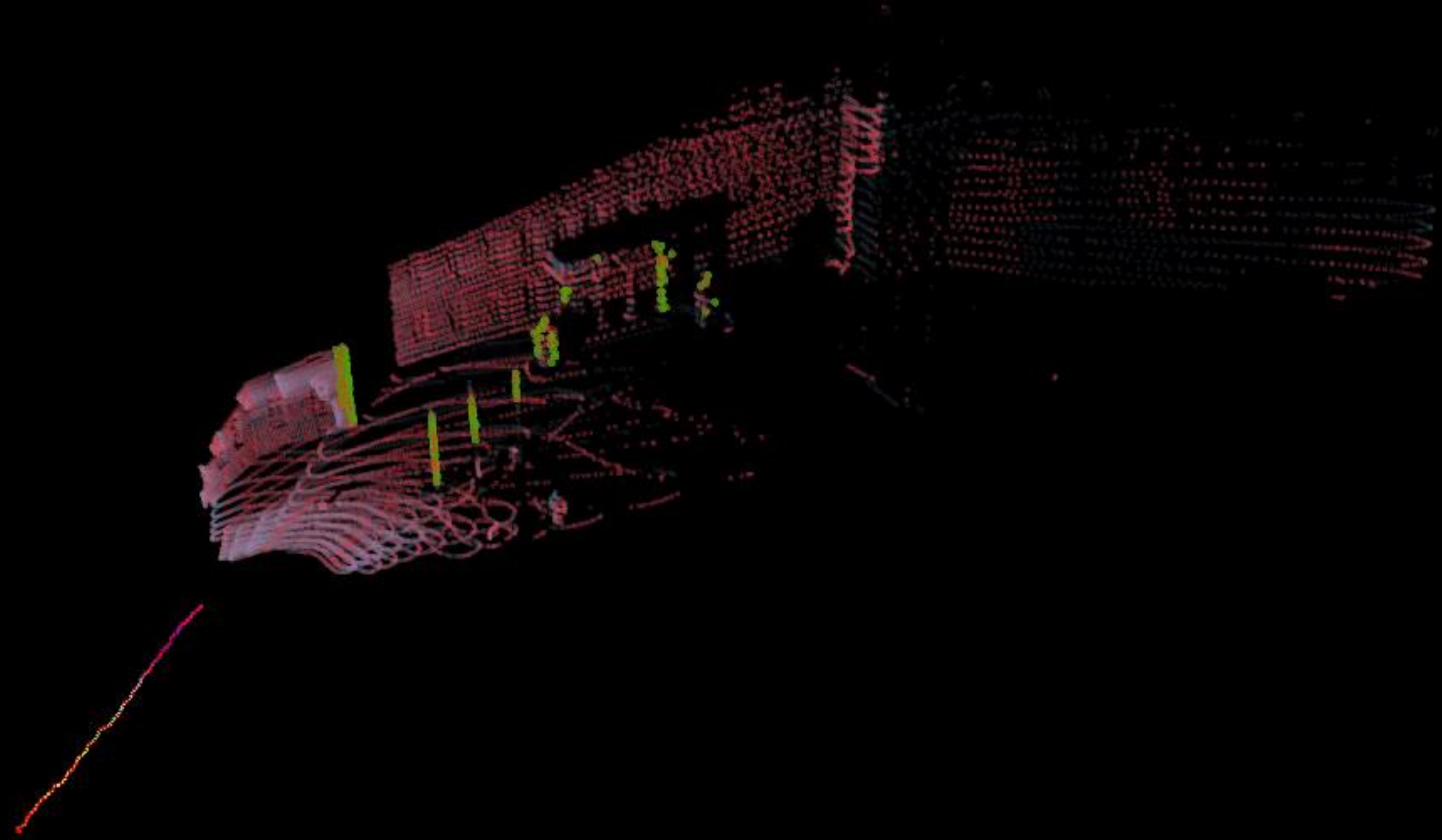
Scan Pattern





Feature Extraction





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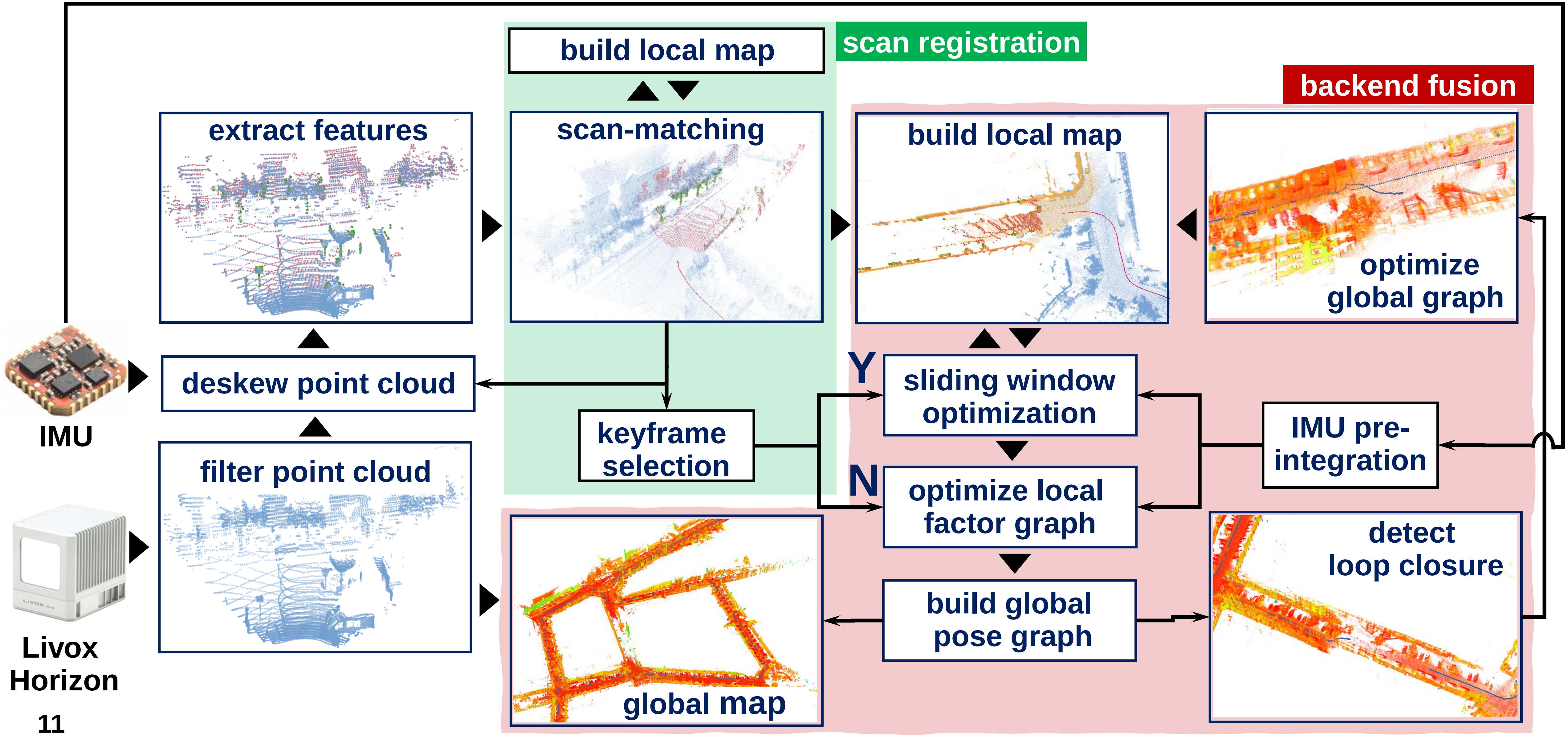
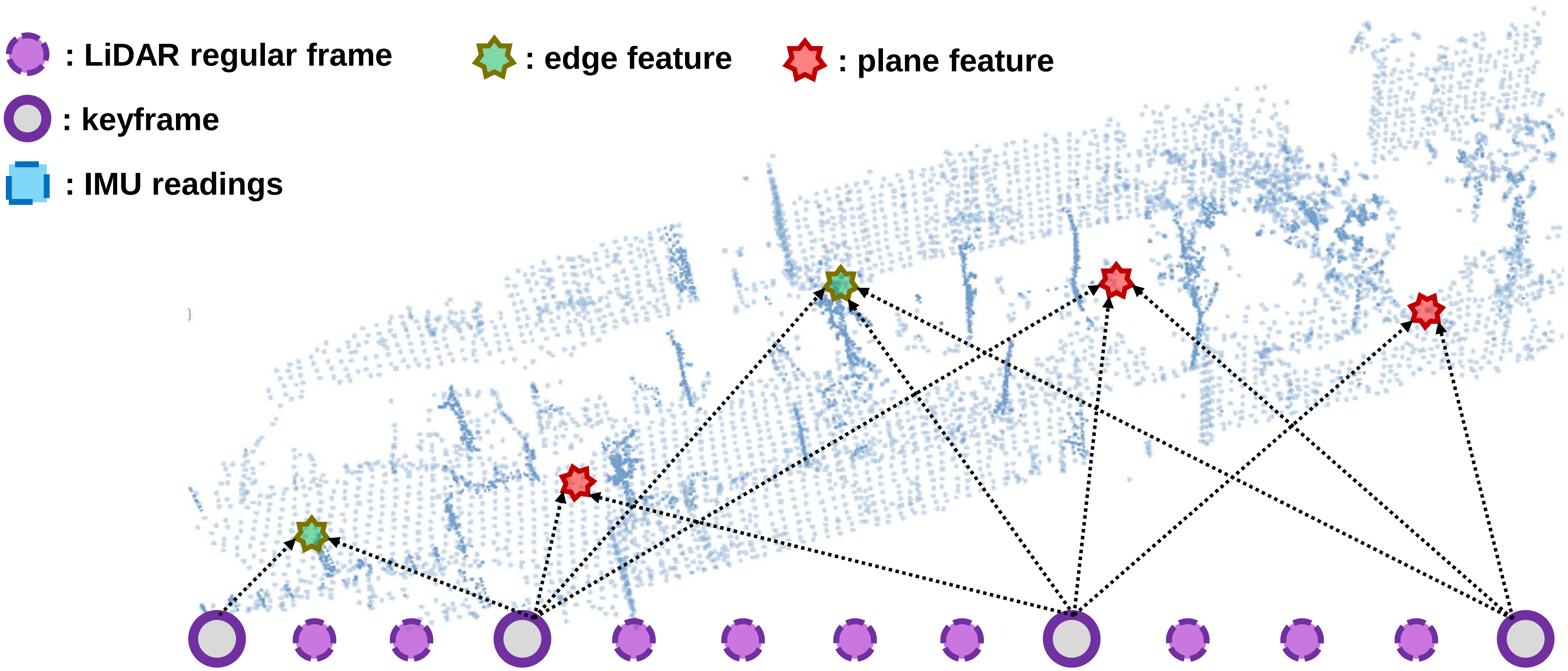
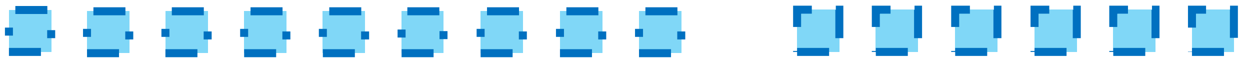


Backend Fusion



state vector: $\check{\mathbf{x}} = \left[\check{\mathbf{t}}^\top, \check{\mathbf{v}}^\top, \check{\mathbf{q}}^\top, \check{\mathbf{b}}^\top \right]^\top \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{S}^3 \times \mathbb{R}^6 \subset \mathbb{R}^{16}$



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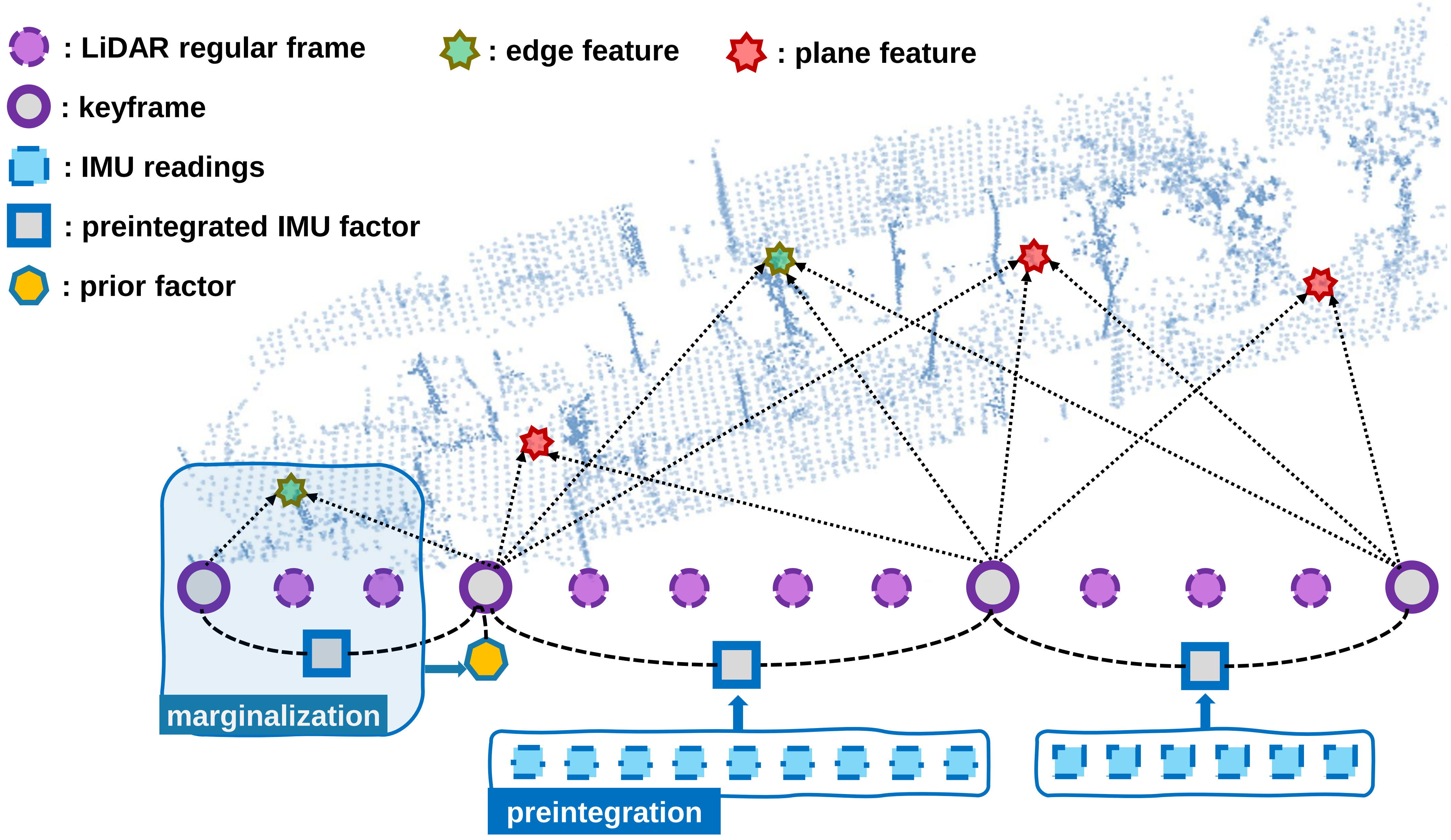
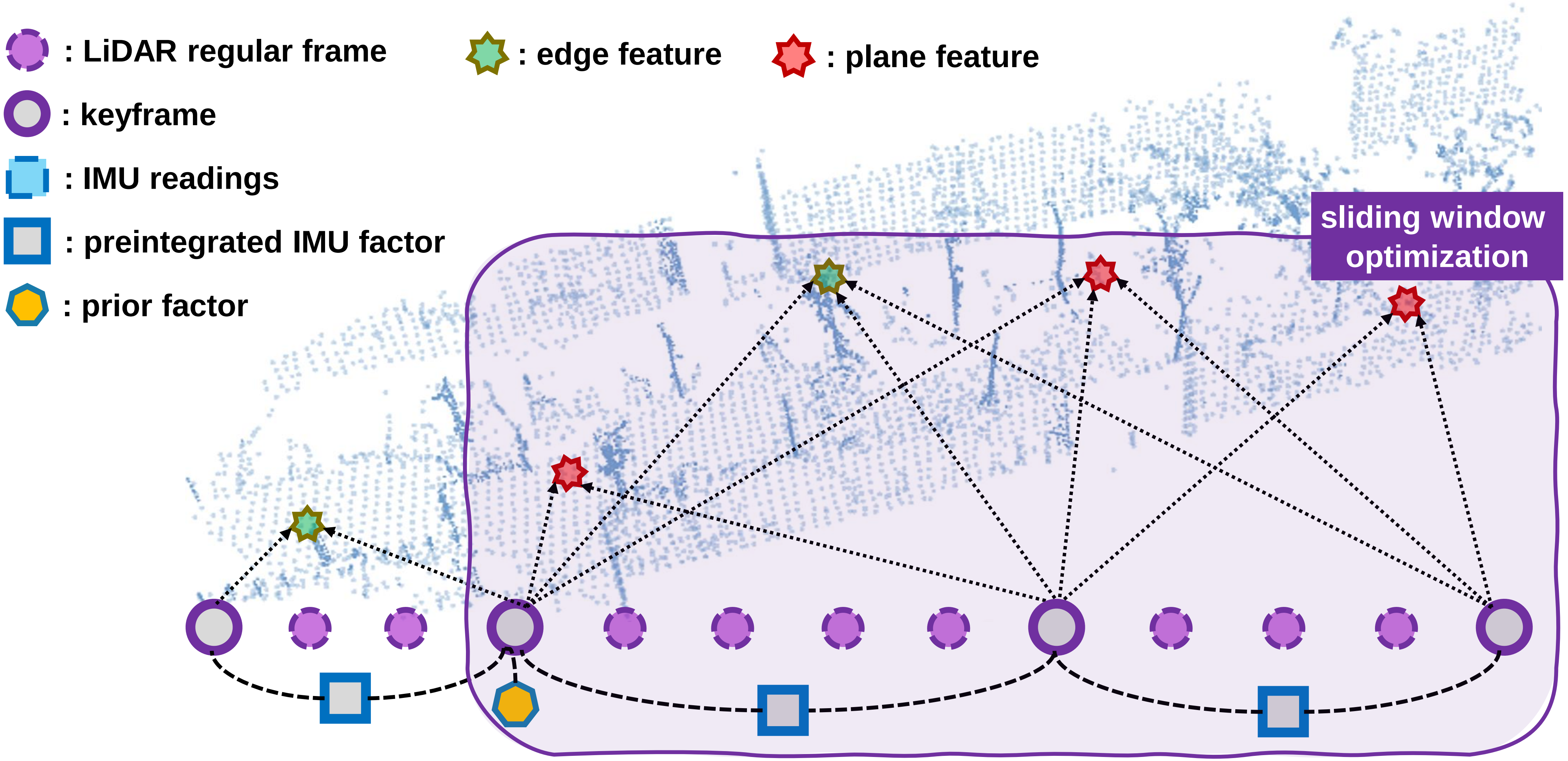


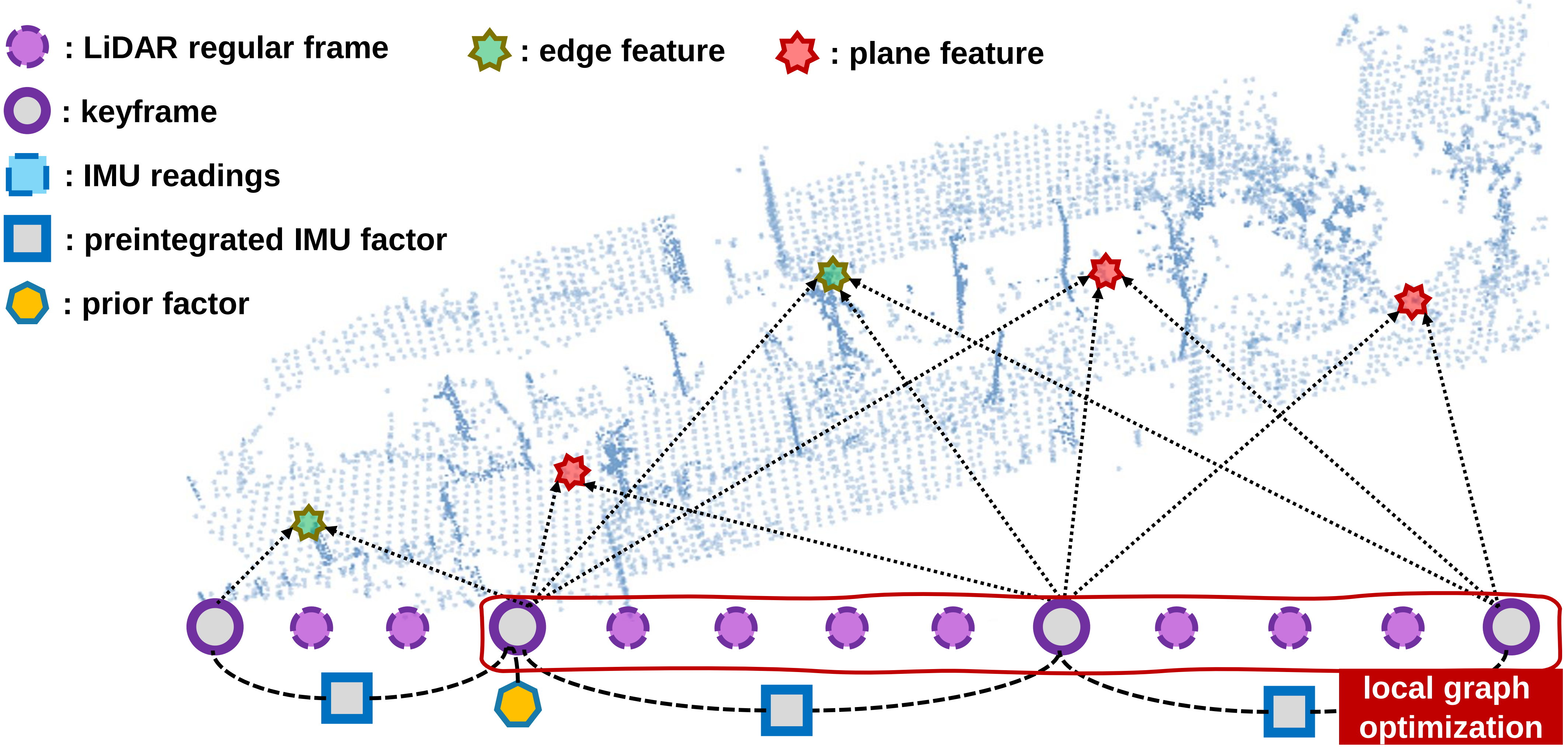
Backend Fusion

Karlsruhe Institute of Technology



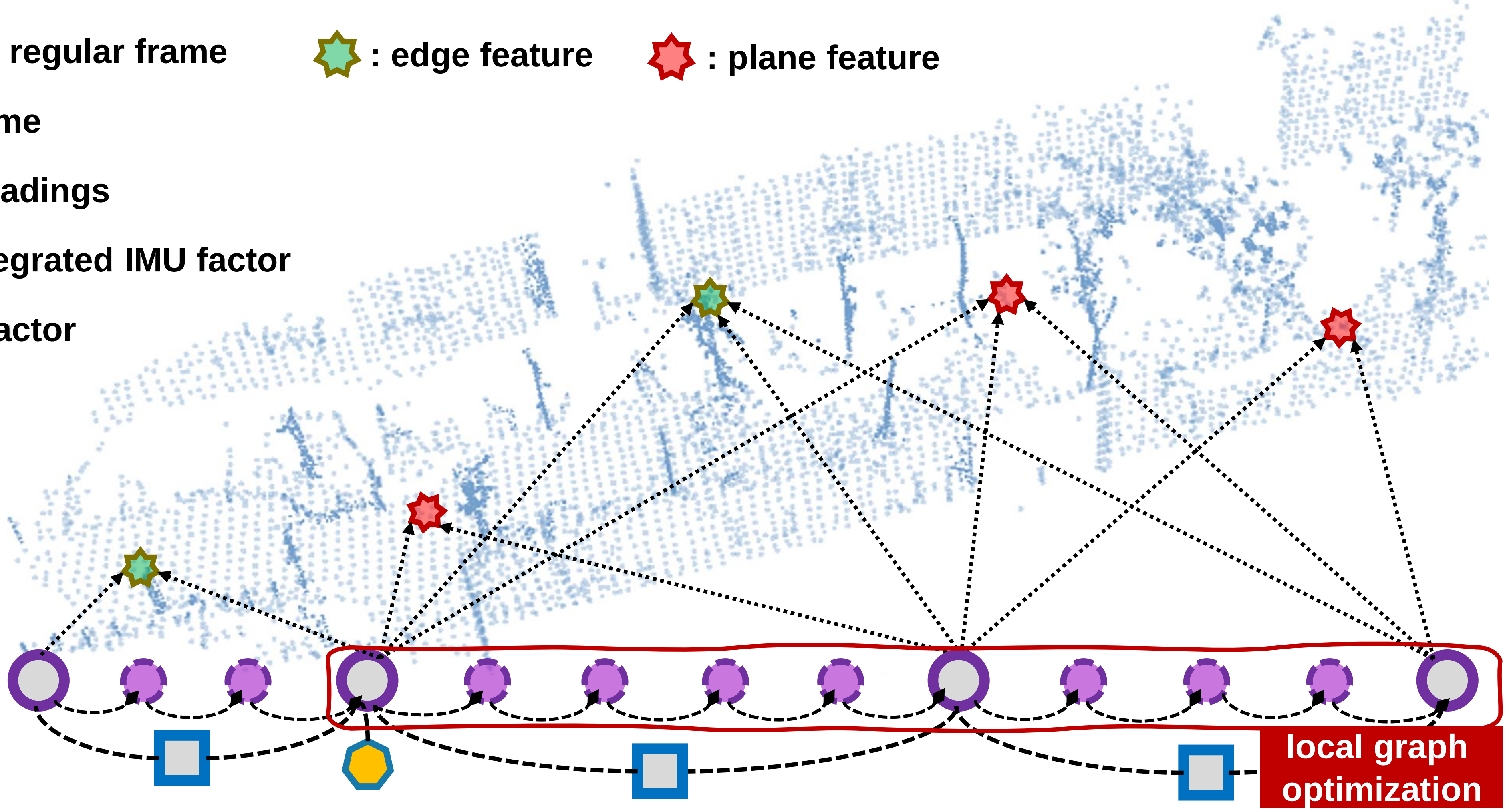
$$\min_{\breve{\mathbf{X}}^{w}} \left\{ \|\mathcal{R}_{\mathrm{P}}(\breve{\mathbf{X}}^{w})\|^{2} + \sum_{k=1}^{\tau_{\mathrm{w}}} \mathcal{J}_{\mathrm{L}}(\breve{\mathbf{x}}_{k}^{w}) + \sum_{k=1}^{\tau_{\mathrm{w}}} \mathcal{J}_{\mathrm{I}}(\breve{\mathbf{x}}_{k}^{w}) \right\}$$

Backend Fusion

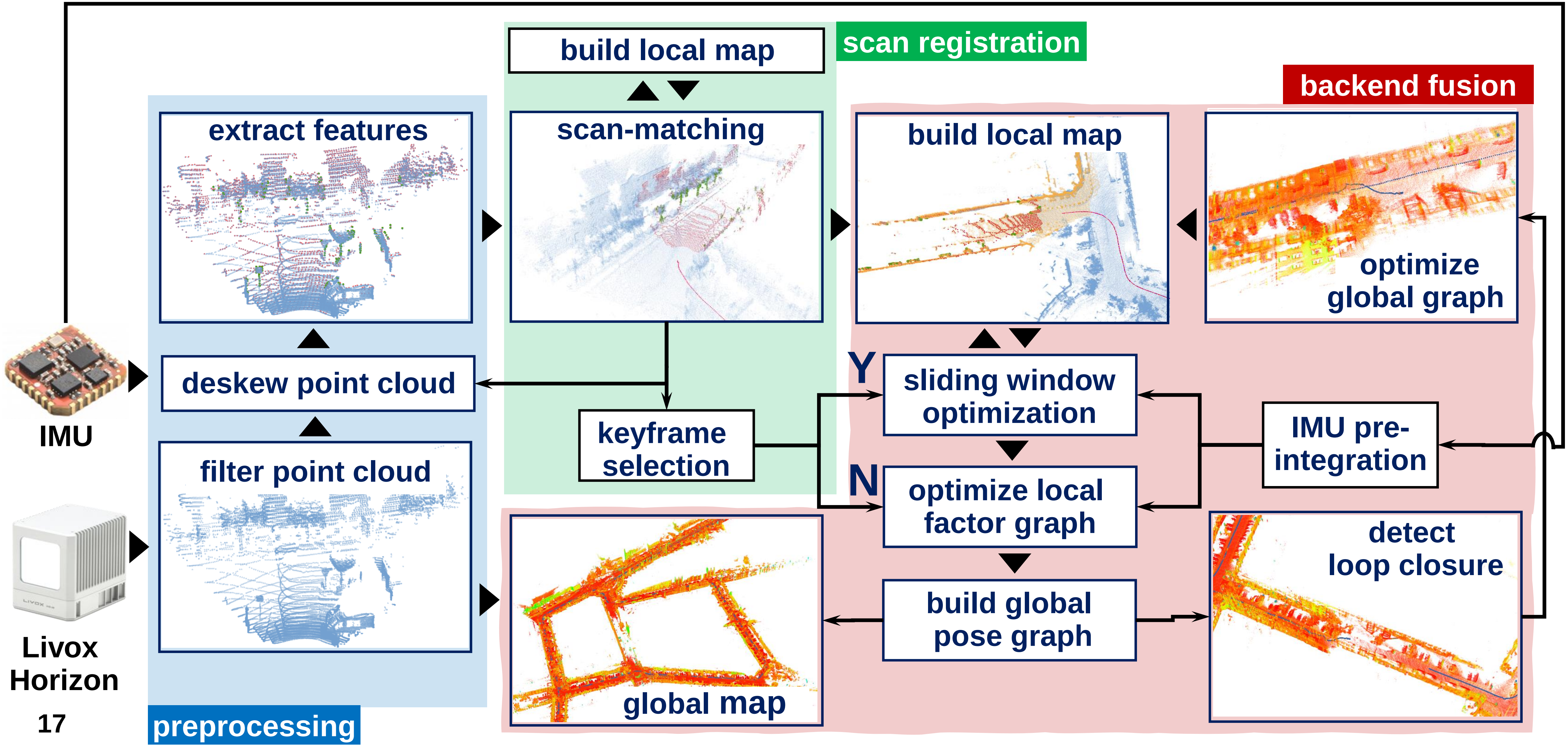


Backend Fusion

- : LiDAR regular frame
- : keyframe
- : IMU readings
- : preintegrated IMU factor
- : prior factor
- : edge feature
- : plane feature



System Pipeline



Sequence FR-IOSB-Long

8x

LiLi-OM-ROT (HDL-64E)

LiLi-OM (Horizon)

Conclusions

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Limitations

- less robust under highly dynamic motions
- limited field of view for single solid-state LiDAR
- simple loop closure detection

Outlook

Conclusions

Karlsruhe Institute of Technology

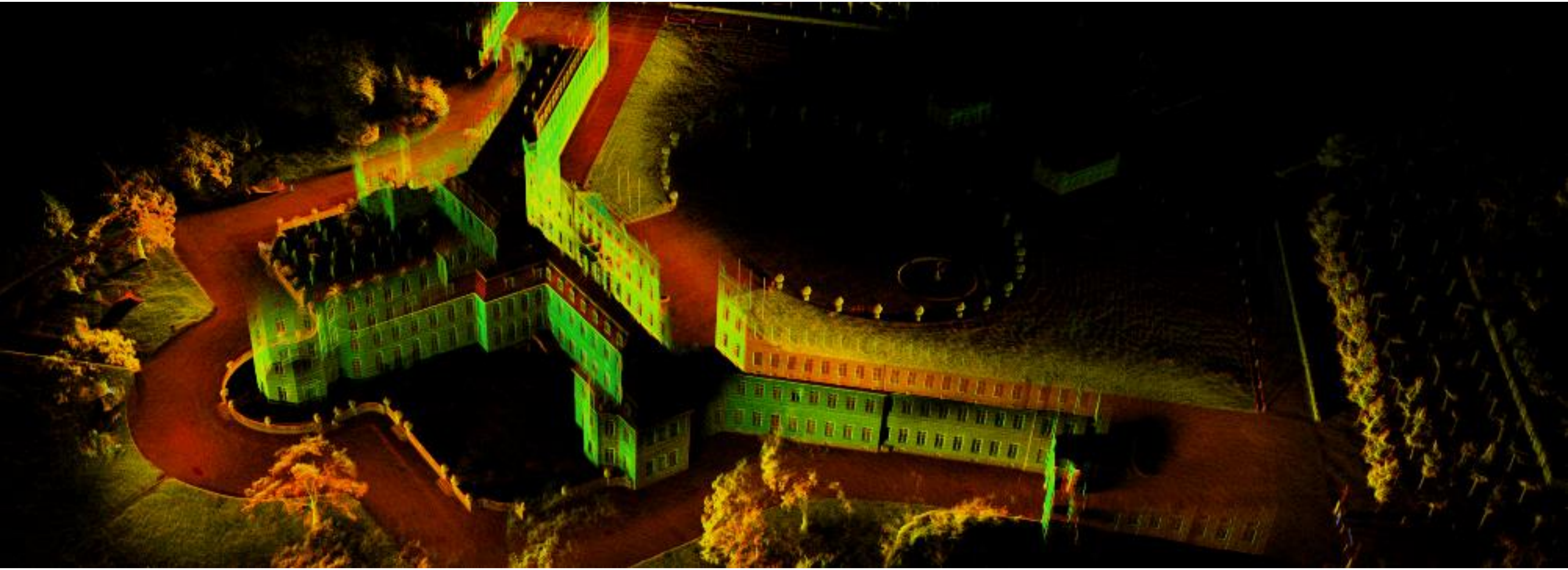
Limitations

- less robust under highly dynamic motions
- limited field of view for single solid-state LiDAR
- simple loop closure detection

Outlook

- fusion of more extensive sensory modalities
  - multiple LiDARs
  - cameras (monocular)
  - event-based cameras
- aerial platform
- innovate sensor fusion framework

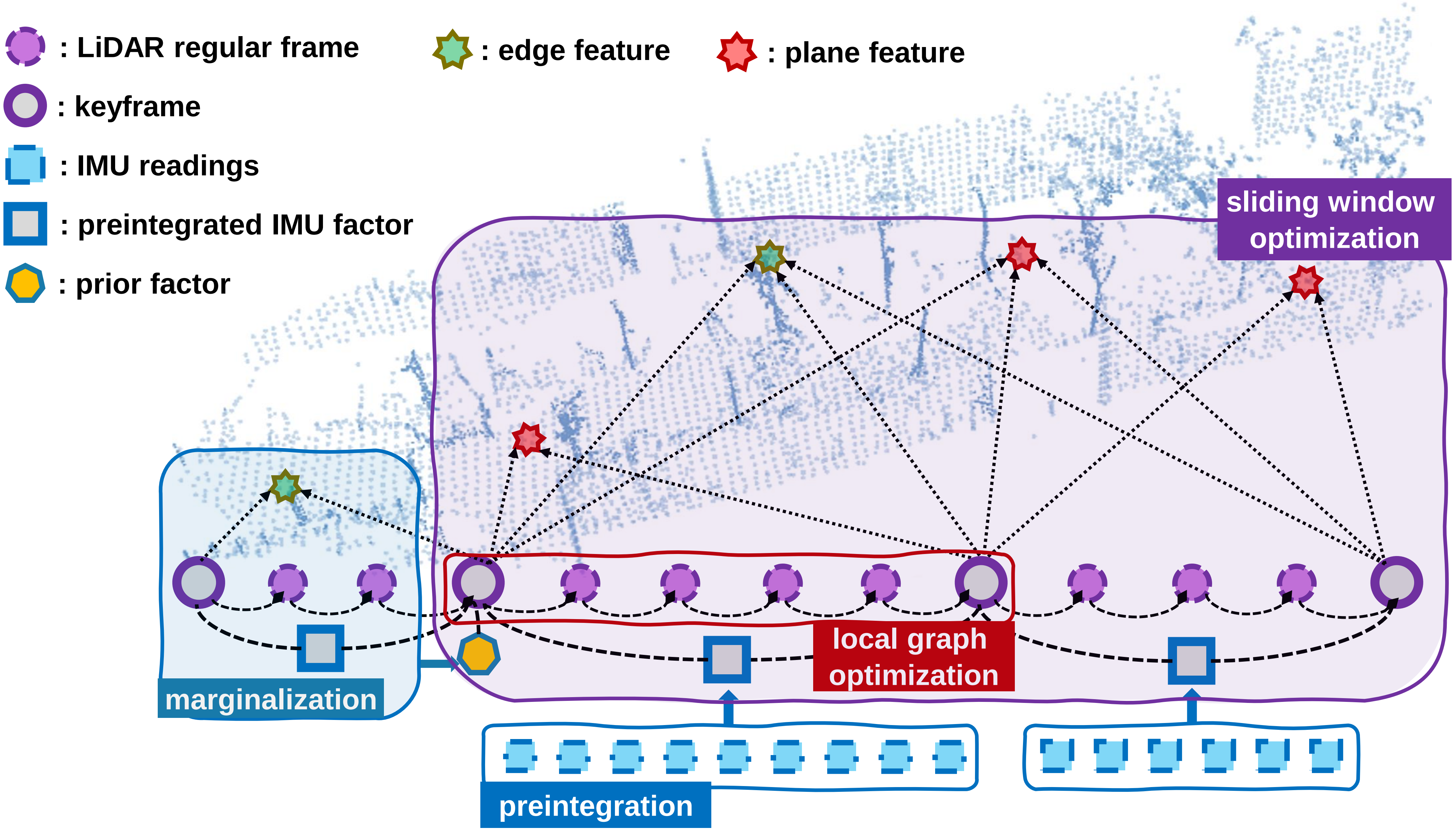
Questions?



Schloss Karlsruhe – A 3D Reconstruction

made by LiLi-OM at https://github.com/KIT-ISAS/lili-om

Backend Fusion



Karlsruhe Institute of Technology

Intelligent Sensor-Actuator-Systems

Iterative Closest Point

Dr.-Ing. Kailai Li

LMA-Exercise 2 | May 09, 2022

Iterative Closest Point (ICP)

- computes spatial transformations between two point clouds by minimizing a distance metric
- fundamental technique for egomotion estimation and mobile perception

Reference: https://www.livoxtech.com/

Iterative Closest Point (ICP)

Reference: Izadi, Shahram, et al. "Kinectfusion: Real-time Dynamic 3D Surface Reconstruction and Interaction." *ACM SIGGRAPH 2011*.

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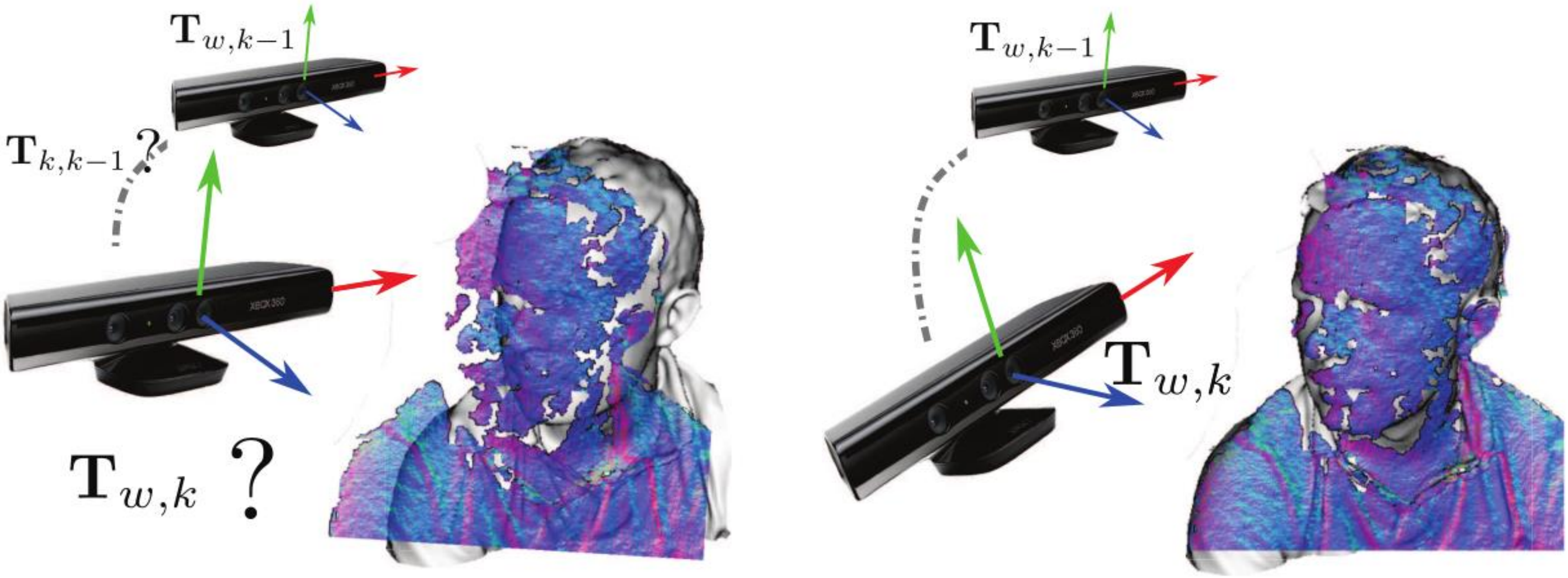
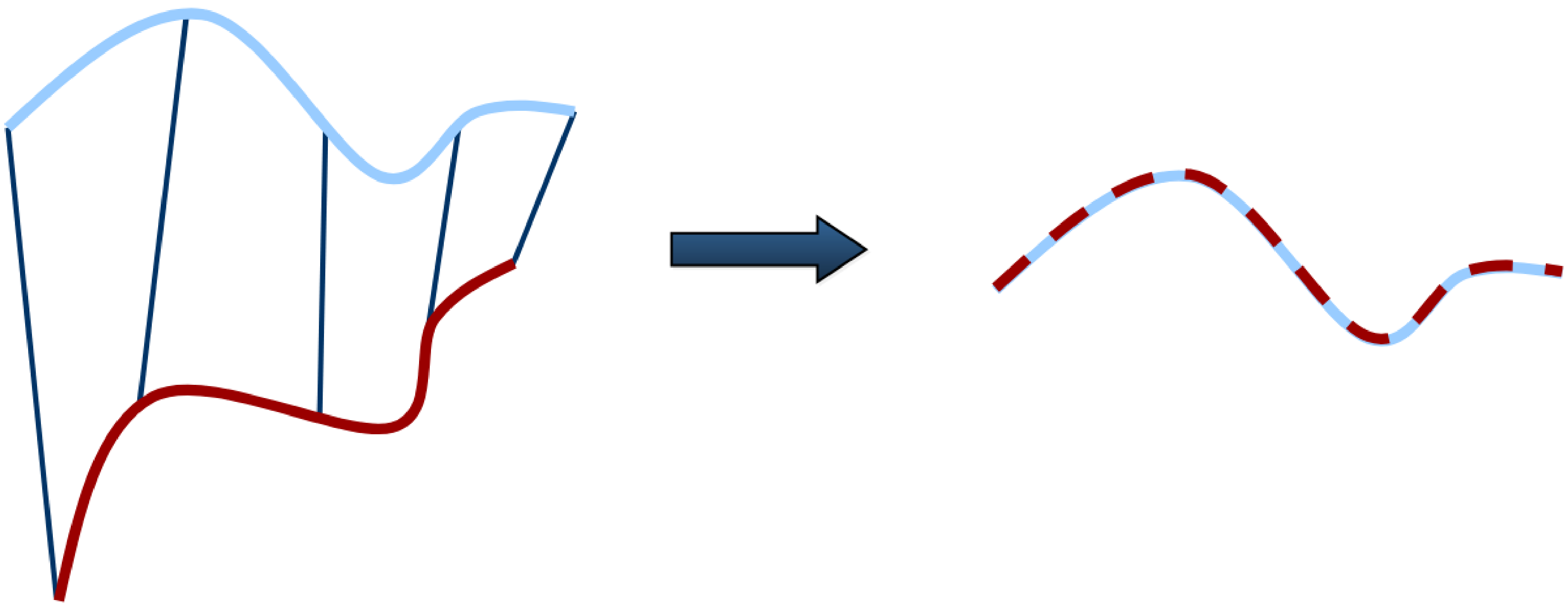


Reference: Izadi, Shahram, et al. "Kinectfusion: Real-time Dynamic 3D Surface Reconstruction and Interaction." *ACM SIGGRAPH 2011*.

Iterative Closest Point (ICP)

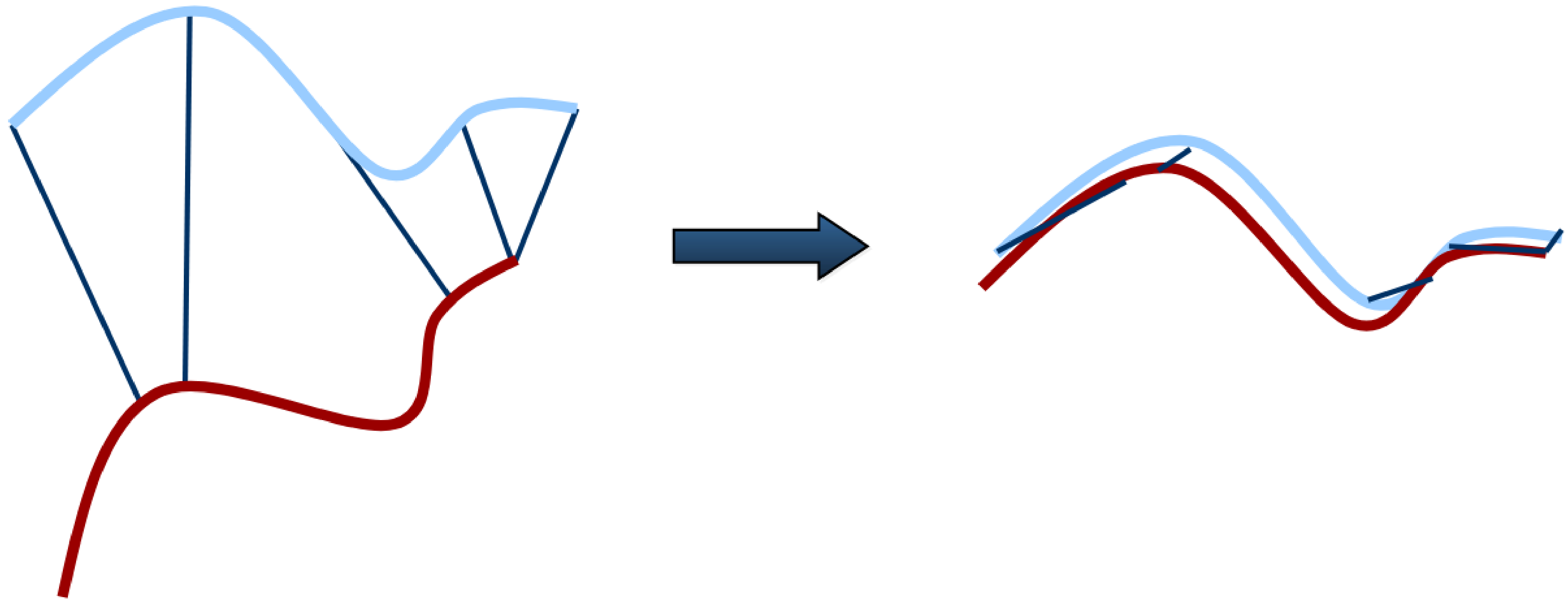
- If the correct correspondences are **known**, the correct relative transformation of SE(3) group can be calculated in closed form via SVD (Singular Value Decomposition)



Courtesy: C. Stachniss *et al.*

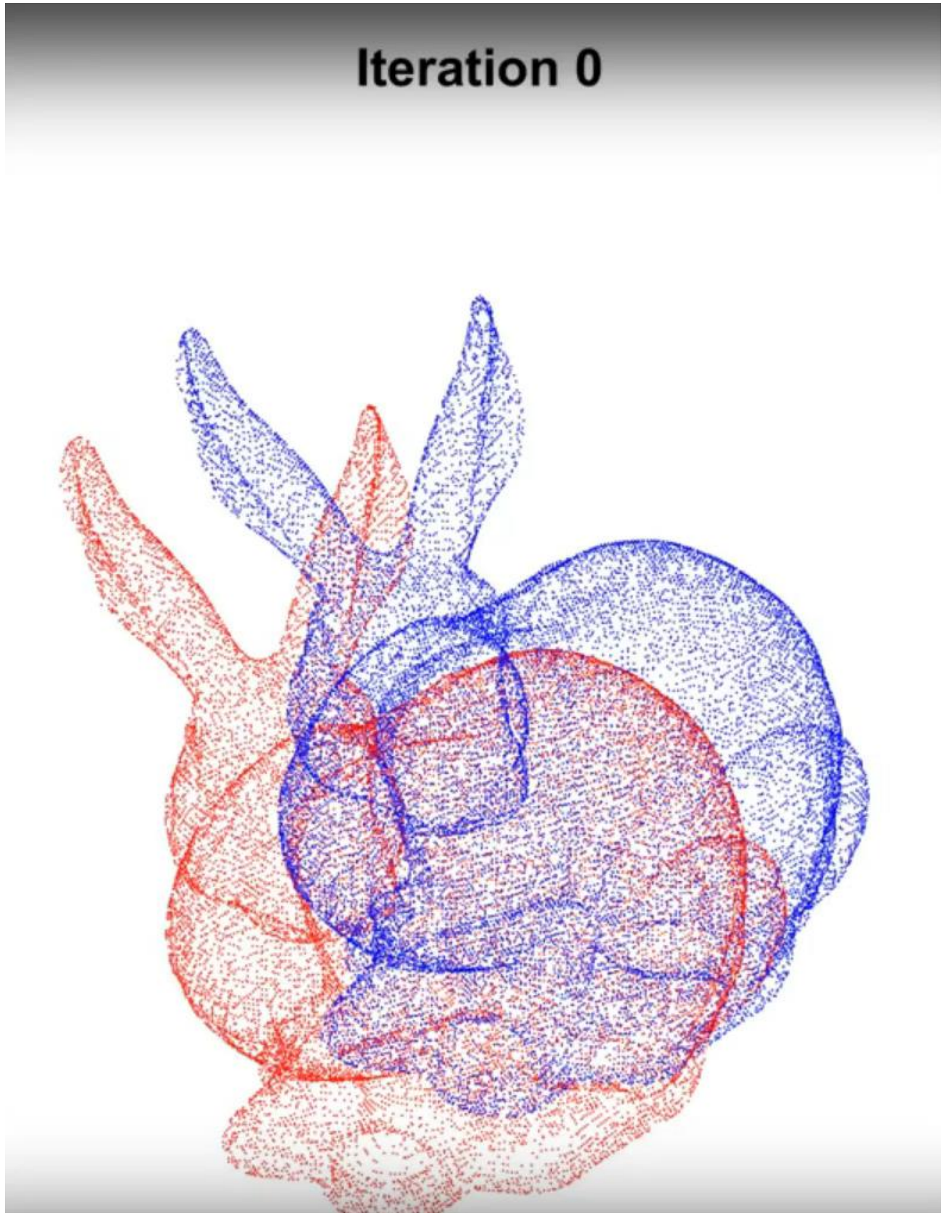
Iterative Closest Point (ICP)

- If the correct correspondences are **unknown**, it is in general impossible to determine the optimal relative rotation and translation in one single step



Courtesy: C. Stachniss *et al.*

Iterative Closest Point (ICP)



- basic idea: iterate to find the alignment
- converges if starting positions are “close enough”

Courtesy: https://www.youtube.com/shorts/uzOCS_gdZuM.

Iterative Closest Point (ICP)

Karlsruhe Institute of Technology

Basic ICP algorithm

1. determine corresponding points
2. compute rotation and translation via SVD
3. apply transformation $\mathbf{T}_{i-1}^{i}$ to the points of the set to be registered
4. compute error metric $\mathcal{D}(\mathbf{T}_{i-1}^{i})$
5. if error decreased and $\mathcal{D} > \epsilon$
   - repeat former steps 1 – 4
   - otherwise stop and output final alignment

Courtesy: C. Stachniss *et al.*

Iterative Closest Point (ICP)

Karlsruhe Institute of Technology

ICP variants

- point subsets (from one or both sets)
- data association
- rejecting certain point pairs (outliers)

Iterative Closest Point (ICP)

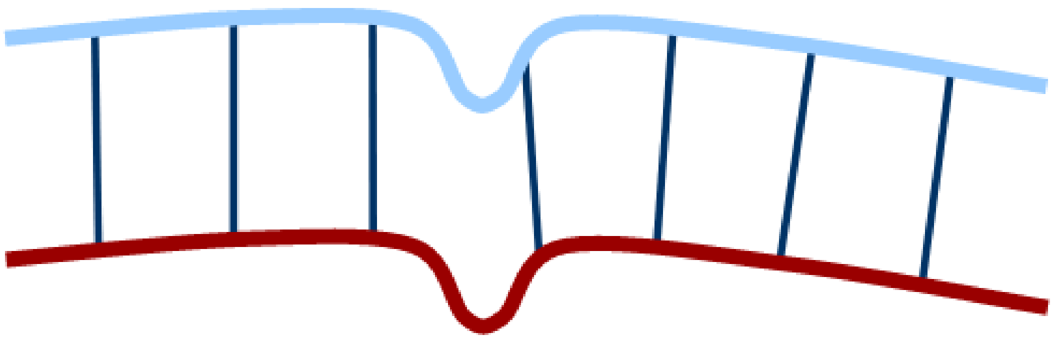
ICP variants: Selecting source points

- use all points
- uniform sub-sampling
- random sampling
- feature-based sampling
- normal-space sampling

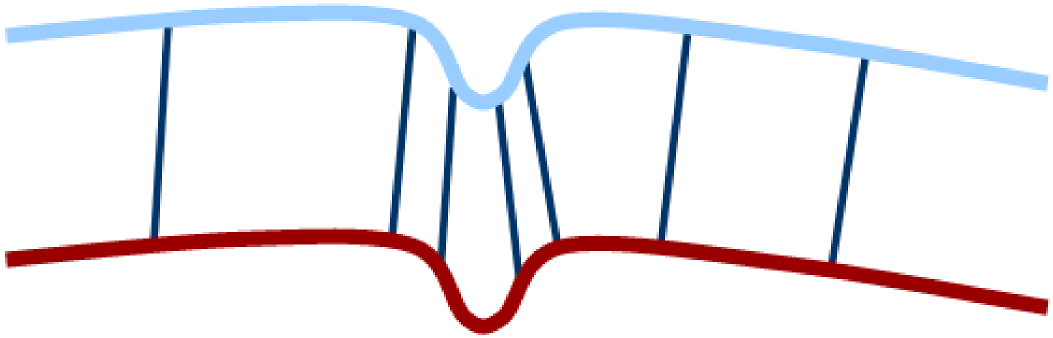
Iterative Closest Point (ICP)

Normal-space sampling

- ensure that samples have normals distributed as uniformaly as possible
- better for mostly smooth areas with sparse features



uniform sampling



normal-space sampling

Courtesy: C. Stachniss *et al.*

Iterative Closest Point (ICP)

Normal-space sampling

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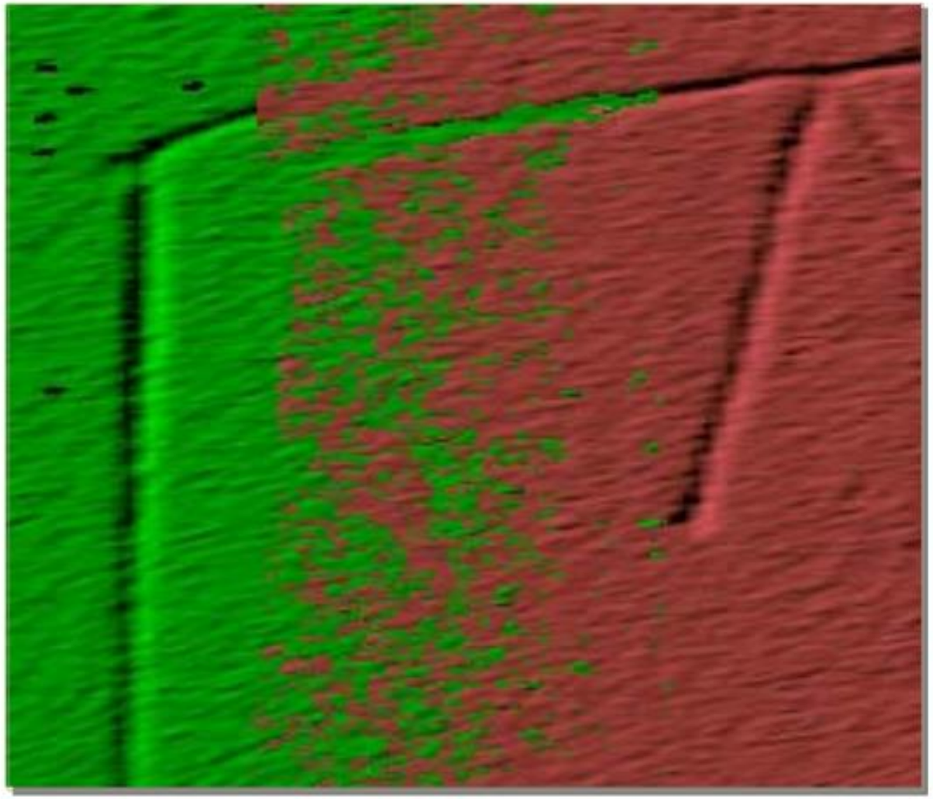
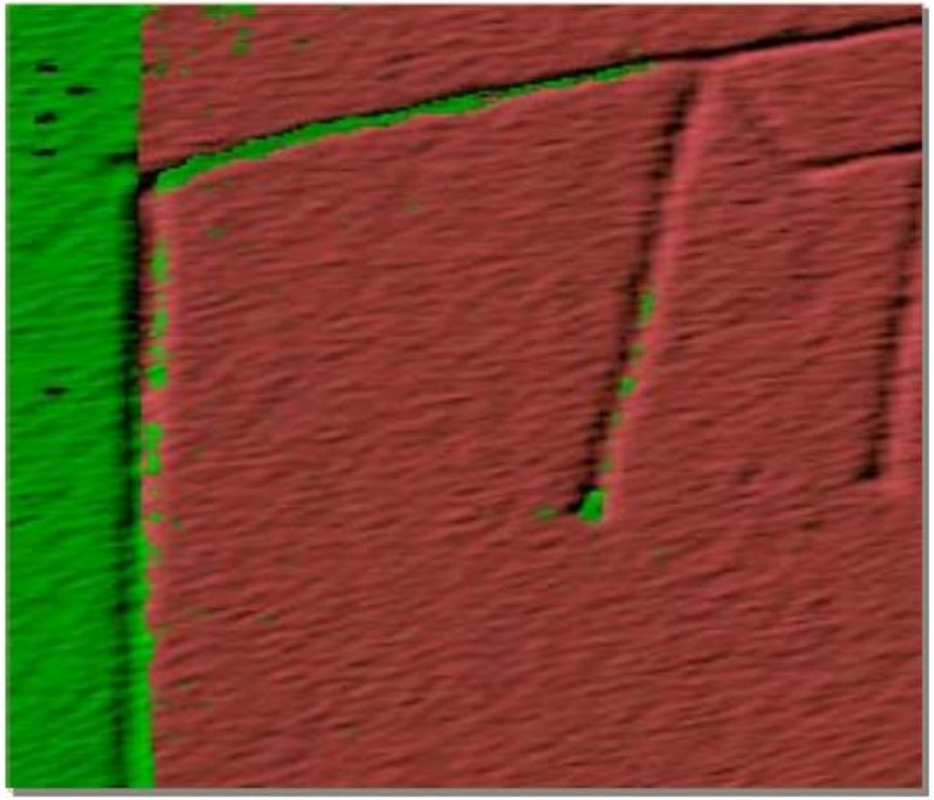


random sampling



normal-space sampling

Courtesy: C. Stachniss *et al.*

Iterative Closest Point (ICP)

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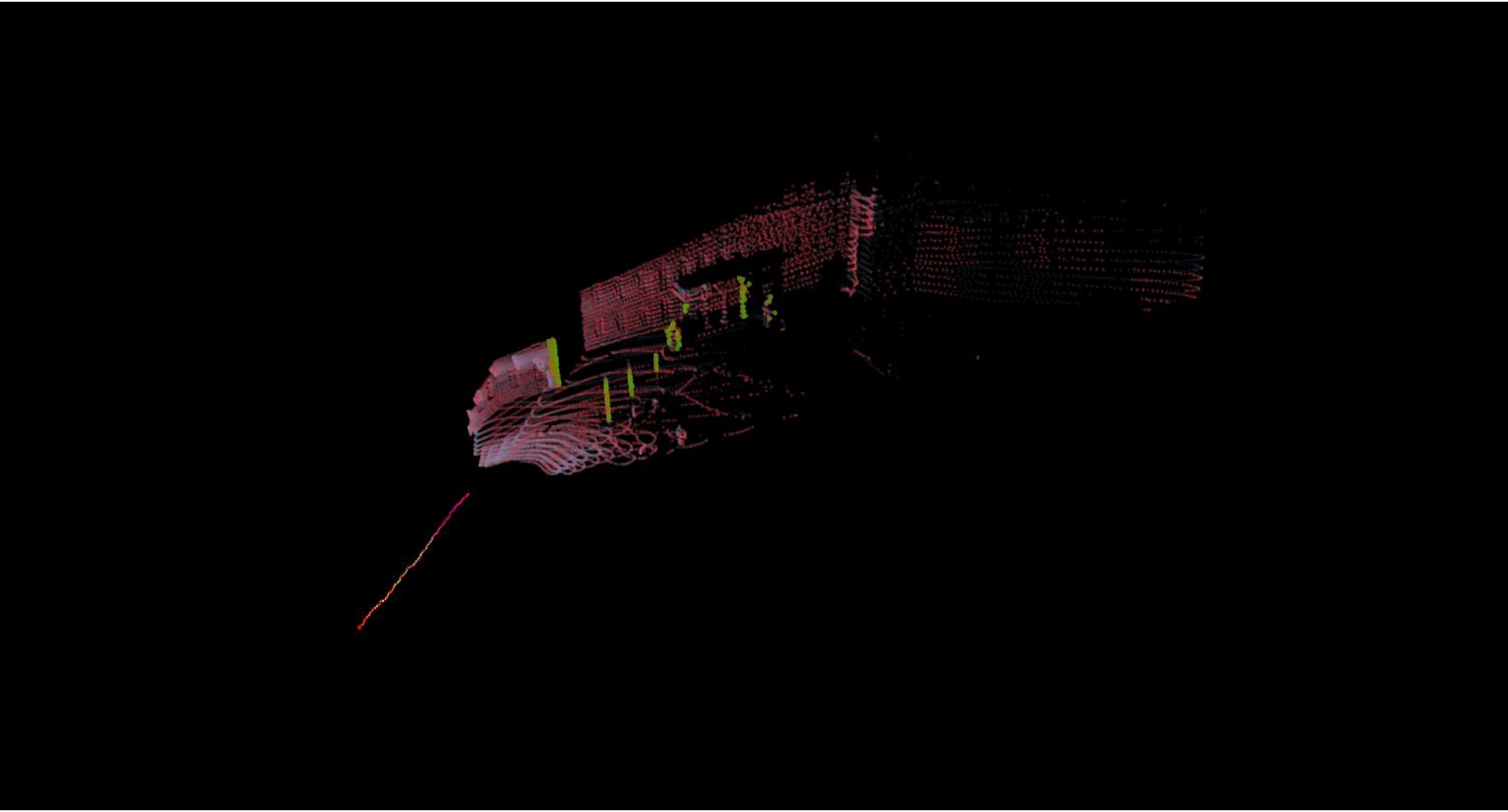
Feature-based sampling

- find more representative points via preprocessing
- better efficiency and accuracy for ICP

Courtesy: C. Stachniss *et al.*



Iterative Closest Point (ICP)

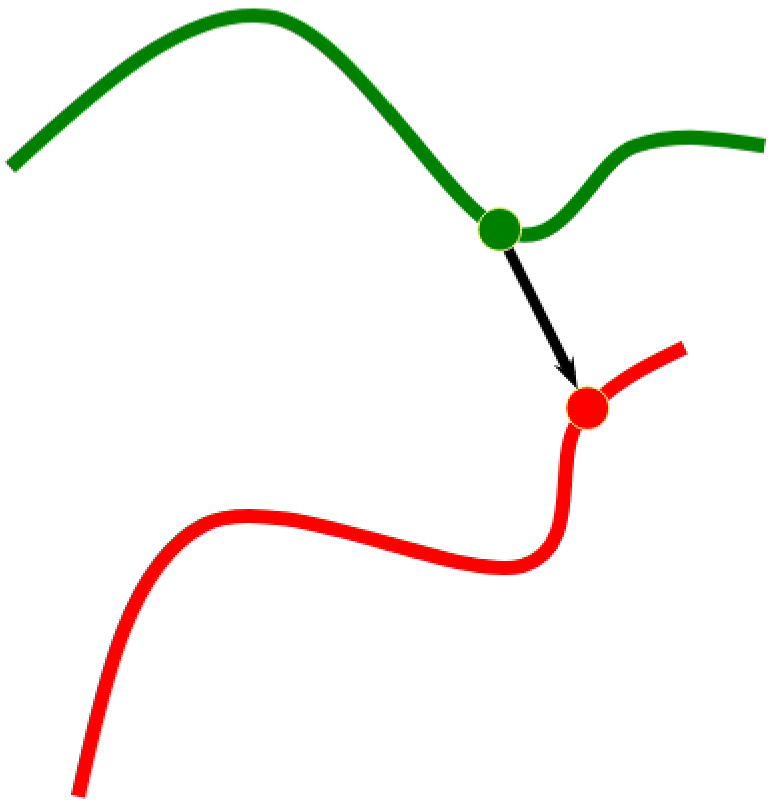
ICP variants: Data association

- has the greatest effect on convergence and speed
- matching approaches:
  - closest points
  - normal shooting
  - closest compatible point

Iterative Closest Point (ICP)

Data association: Closest point

- in general stable
- slow convergence

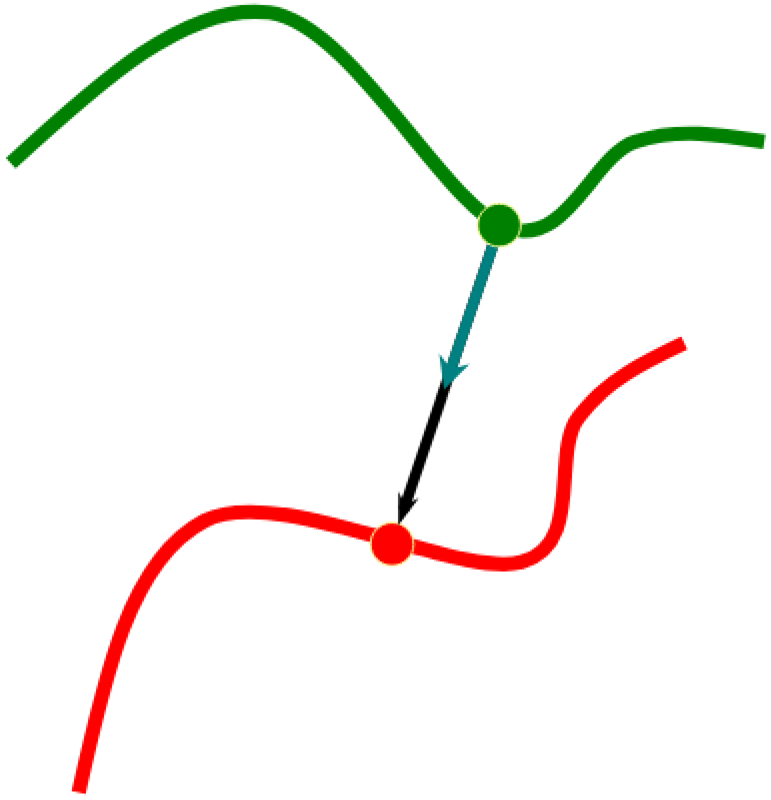


Courtesy: C. Stachniss *et al.*

Iterative Closest Point (ICP)

Data association: Normal shooting

- slightly better convergence than closest point for smooth structures
- worse for noisy or complex structure

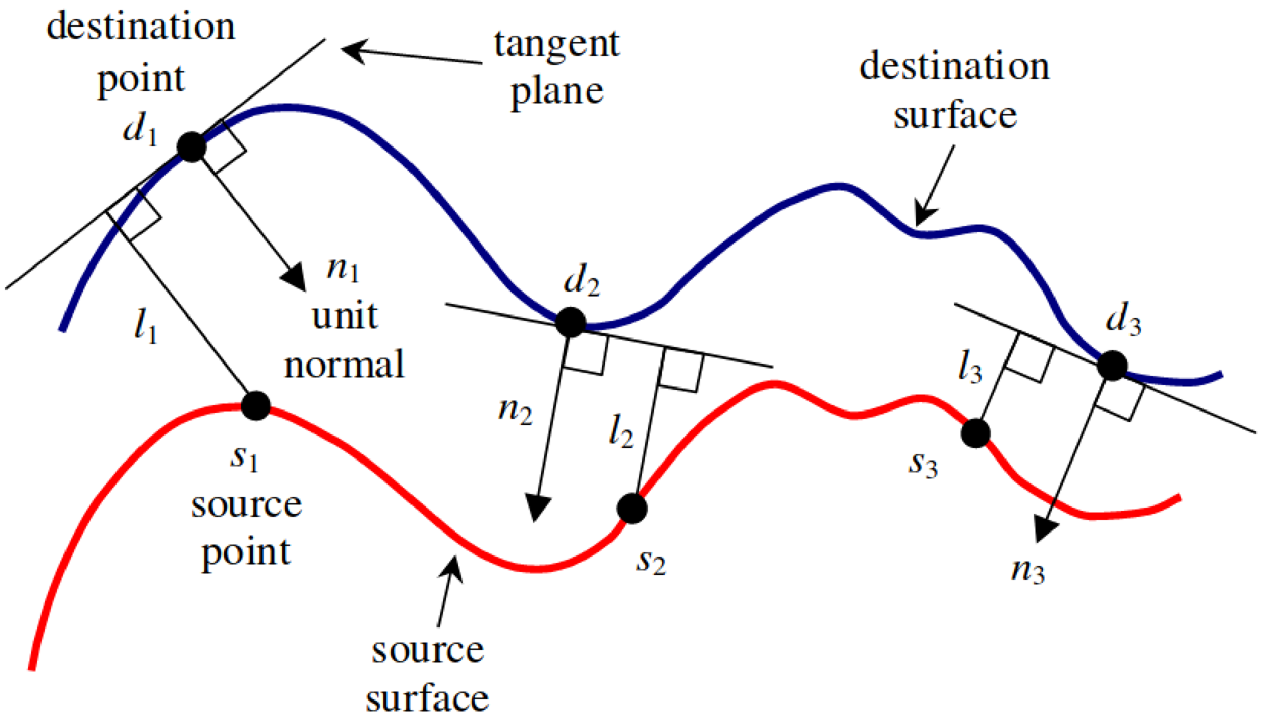


Courtesy: C. Stachniss *et al.*

Iterative Closest Point (ICP)

Data association: Closest compatible point

- considers compatibility of points, e.g., normals, colors, curvature, other local features, etc.
- example: find correspondence by minimizing point-to-plane error metric via standard nonlinear least squres methods (Levenberg-Marquardt algorithm)

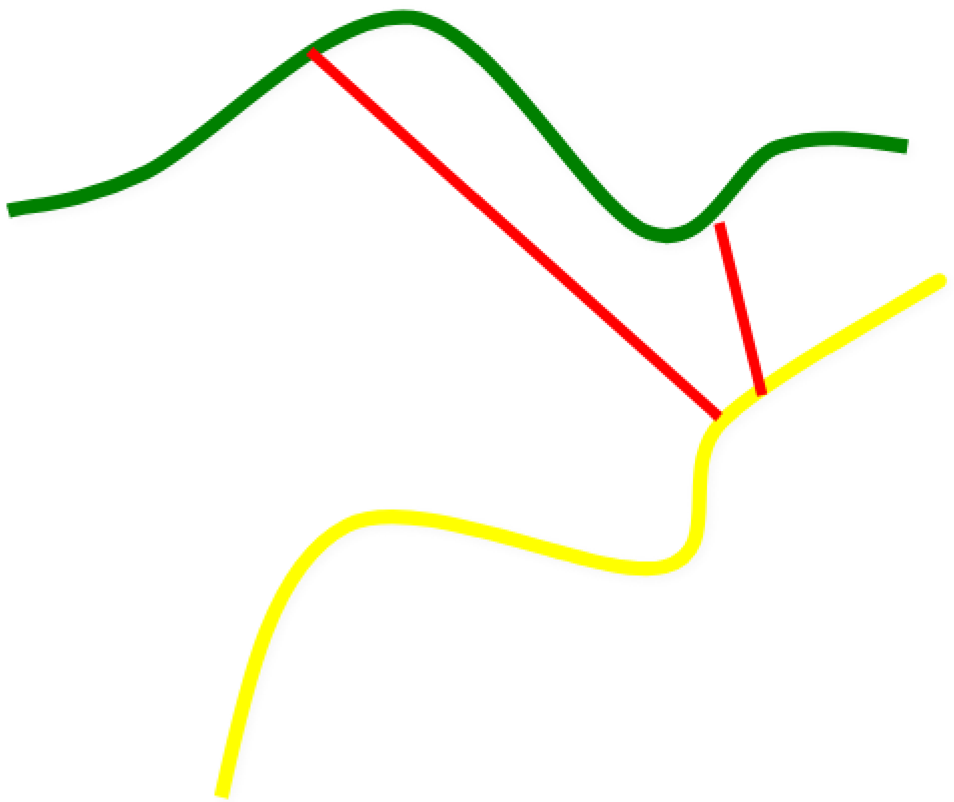


Courtesy: C. Stachniss *et al.*

Iterative Closest Point (ICP)

ICP variants: Rejecting point pairs (outliers)

- corresponding points with point-to-point distance larger than a given threshold
- rejecting pairs that are not consistent with neighboring pairs
- e.g., sort all correspondences w.r.t. their error and delete the worst 1%



Courtesy: C. Stachniss *et al.*

Iterative Closest Point (ICP)

Karlsruhe Institute of Technology

Summary

- ICP is a very powerful technique for robotic localization and perception.
- Major problem is to find correct data associations (accuracy and convergence).
- Given correct data associations, transformations can be computed efficiently via SVD.
- ICP does not always converge.

Courtesy: C. Stachniss *et al.*

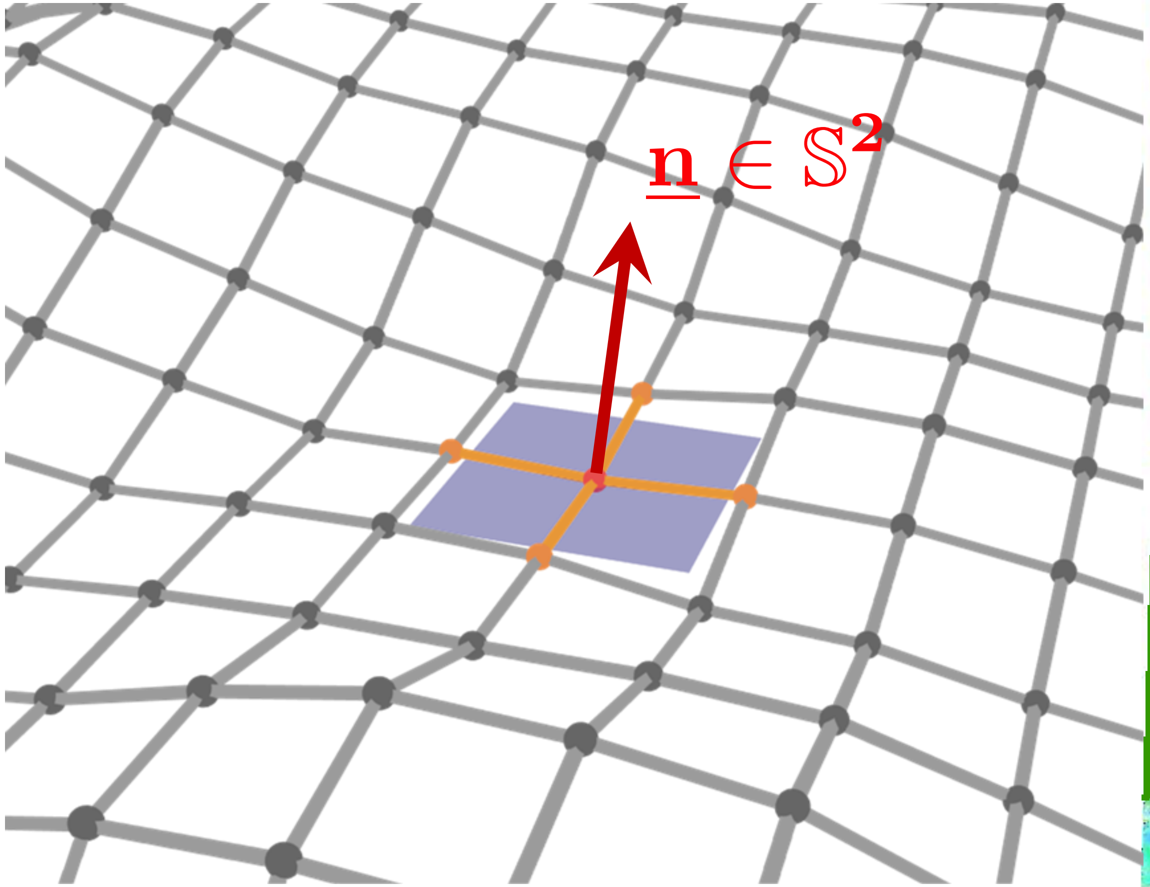
Directional Random Variables

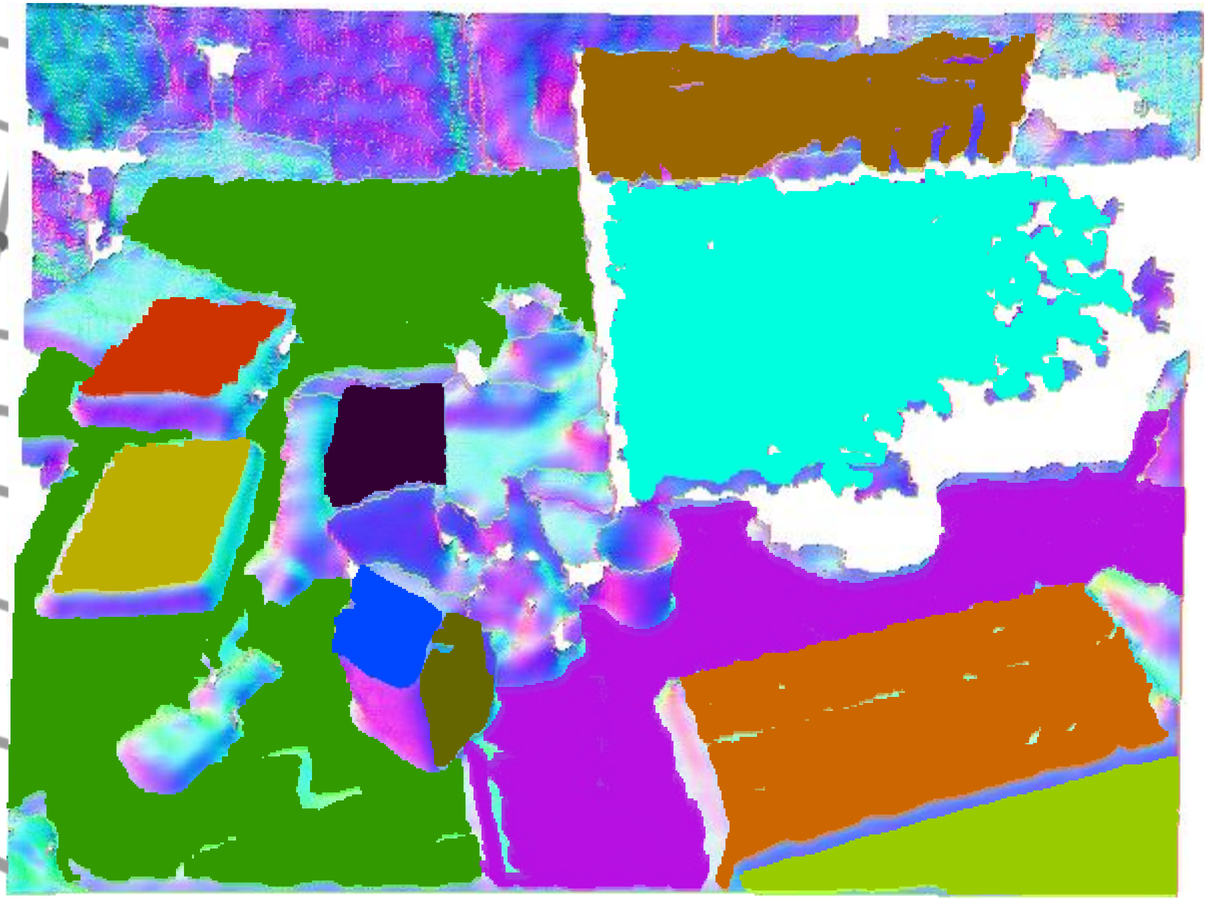
Dr.-Ing. Kailai Li

LMA-Exercise 3 | May 16, 2022

Directional variables are ubiquitous.

- highly parallelized plane extraction on depth images



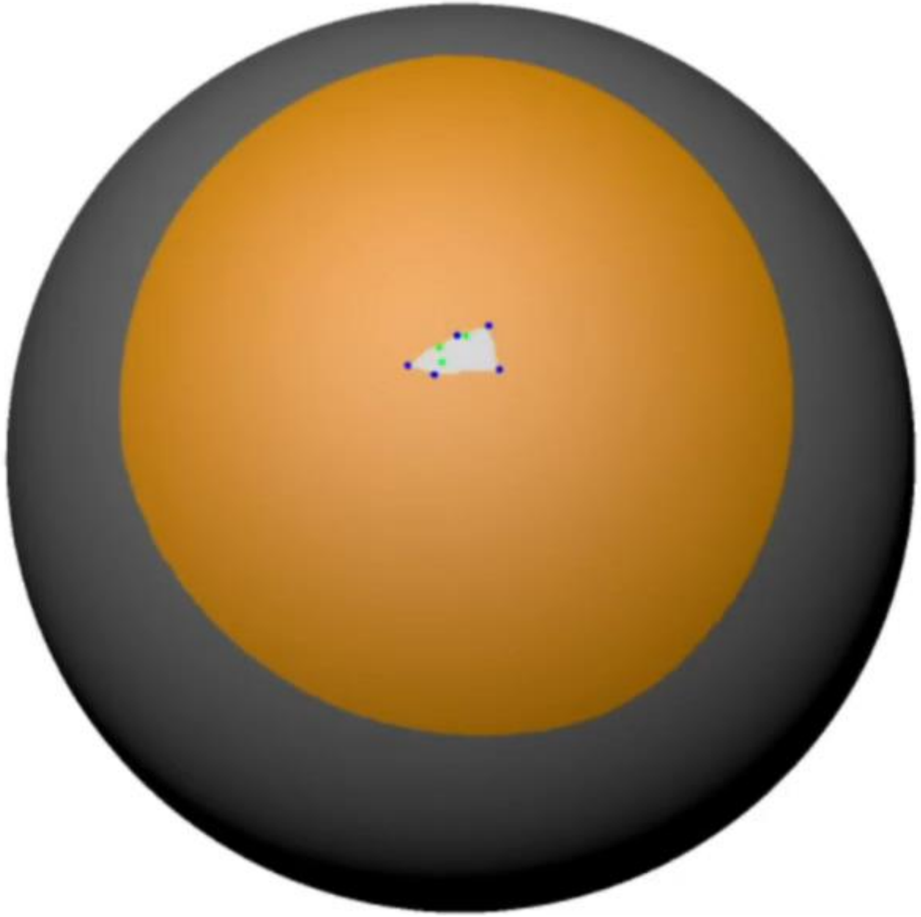


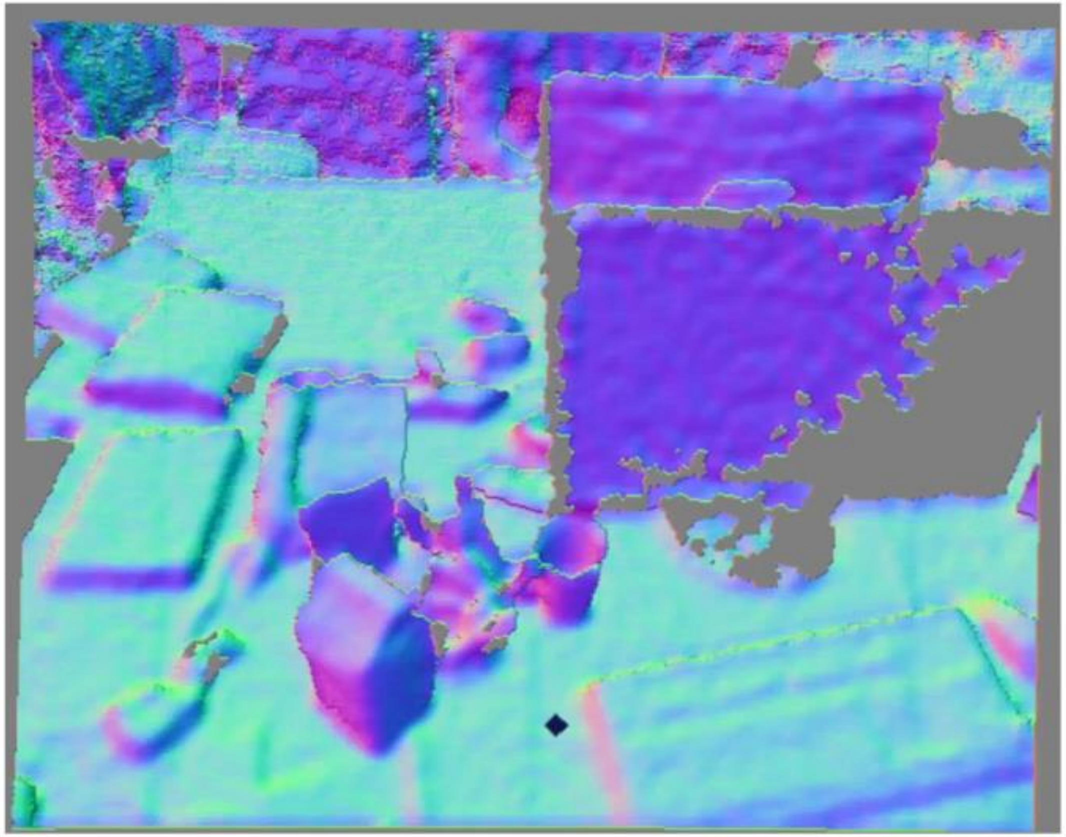
$$\mathbb{S}^2 = \{\underline{x} \in \mathbb{R}^3 \mid \|\underline{x}\| = 1\}$$

Hannes Möls, Kailai Li, and Uwe D. Hanebeck. 2020 IEEE International Conference on Robotics and Automation (ICRA'20)

Directional variables are ubiquitous.

- highly parallelized plane extraction on depth images transferred to unit spheres





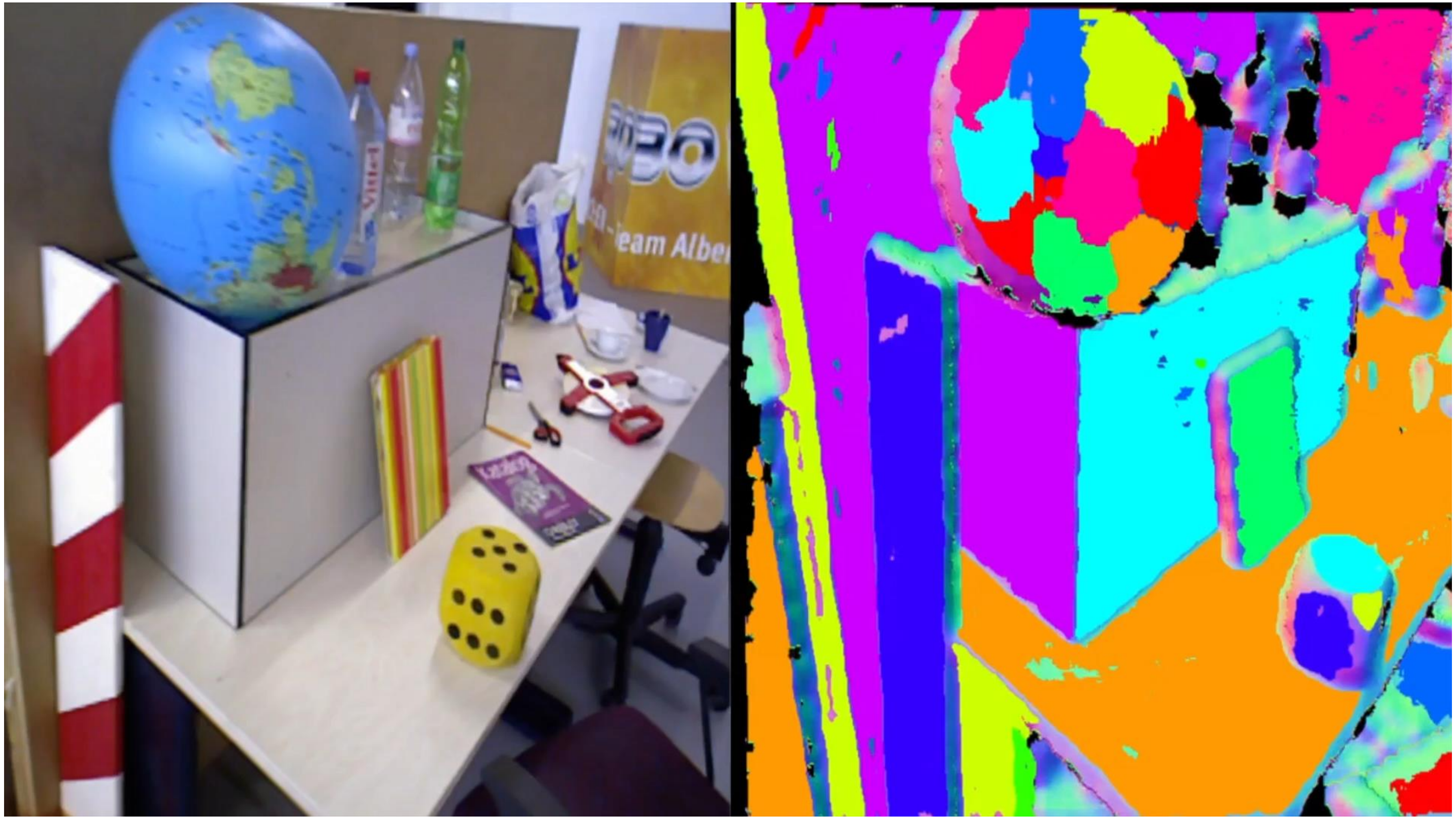
clustering on unit sphere ← segmentation in normal map

Hannes Möls, Kailai Li, and Uwe D. Hanebeck. 2020 IEEE International Conference on Robotics and Automation (ICRA'20)

Directional variables are ubiquitous.

KIT Karlsruhe Institute of Technology

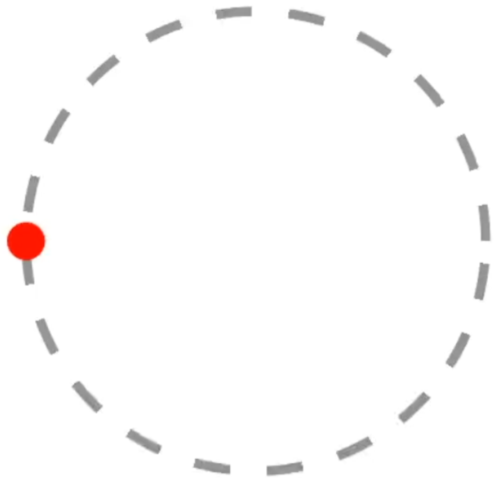
- highly parallelized plane extraction on depth images transferred to unit spheres
  - ➢ full resolution (640 x 480) pixelwise segmentation at 60 Hz on embedded GPU (**fastest ever**)



Hannes Möls, Kailai Li, and Uwe D. Hanebeck. 2020 IEEE International Conference on Robotics and Automation (ICRA'20)

Directional Random Variables

- periodic, nonlinear or symmetric topological structure
- uncertainty quantification
  - conventional scheme → Gaussian model in locally linearized space



local perturbation assumption

↓

deteriorated performance under large uncertainty or fast transition

Directional Random Variables

- periodic, nonlinear topological structure
- uncertainty quantification
  - conventional scheme → Gaussian model in locally linearized space
  - directional statistics → parametric models inherently defined on directional manifolds

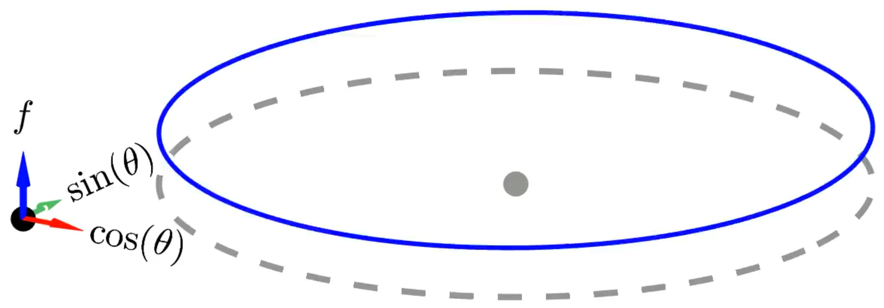
von Mises distribution

$$f_{\mathcal{VM}}(\theta; \alpha, \kappa) = \frac{1}{2\pi \mathcal{I}_0(\kappa)} \cdot \exp\big(\kappa \cos(\theta - \alpha)\big)$$

$\alpha:$ mean

$\kappa:$ concentration

unimodal



$\mathcal{VM}(\boldsymbol{\theta}; \alpha, 0.1)$

Directional Random Variables

Karlsruhe Institute of Technology

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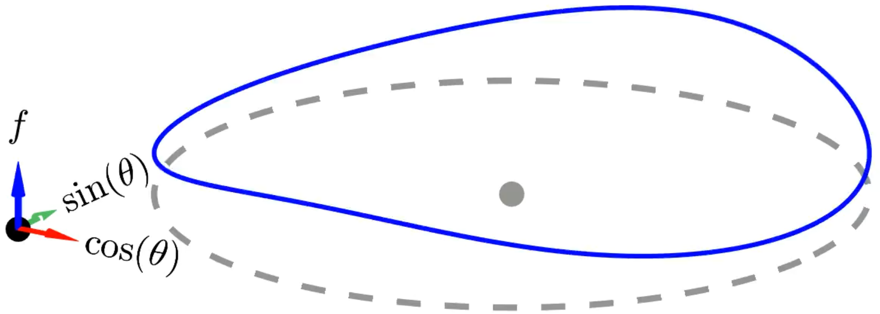

$\mathcal{B}(\underline{\boldsymbol{x}};\ \mathbf{Z}_{1.00},\ \mathbf{M}_{\alpha})$

Bingham distribution

$$f_{\mathcal{B}}(\underline{x}) = \frac{1}{N(\mathbf{Z})} \exp(\underline{x}^\top \mathbf{M}\,\mathbf{Z}\,\mathbf{M}^\top \underline{x})$$

unit circle/sphere/hyperspheres $\mathbb{S}^{d-1} \subset \mathbb{R}^d$

$\mathbf{Z}$: concentration

$\mathbf{M}$: orientation

antipodal symmetry → quaternions

normalization constant

$$N(\mathbf{Z}) = \left|\mathbb{S}^{d-1}\right| \cdot {}_1\mathcal{F}_1\left(1/2, d/2, \mathbf{Z}\right)$$

no closed-form solution

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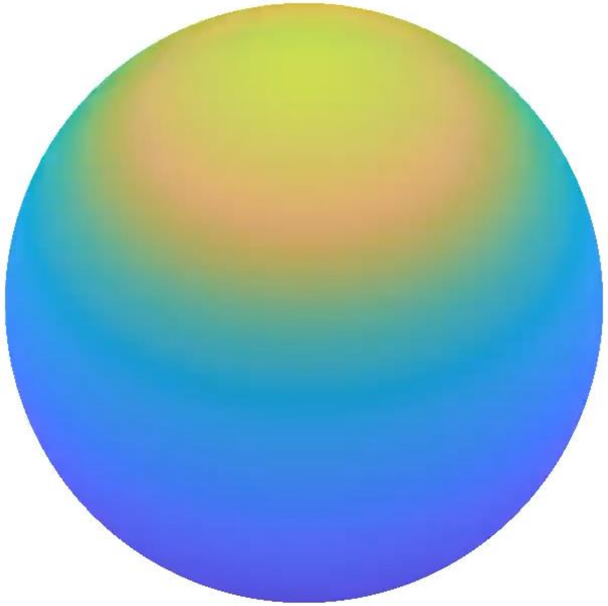


$\mathcal{VMF}(\underline{x}; \underline{\alpha}, 1.00)$

von Mises–Fisher distribution

$$f_{\mathcal{VMF}}(\underline{x}; \underline{\alpha}, \kappa) = N_d(\kappa) \cdot \exp(\kappa\, \underline{\alpha}^\top \underline{x})$$

unit sphere/hyperspheres $\underline{x} \in \mathbb{S}^{d-1} \subset \mathbb{R}^d$

κ : concentration

$\underline{\alpha}$: mean

unimodal, isotropic dispersion

normalization constant

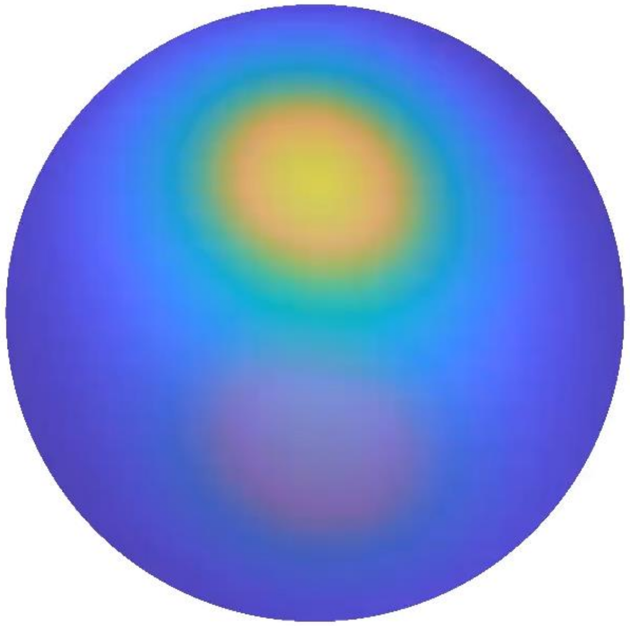
$$N_d(\kappa) = \frac{\kappa^{d/2-1}}{(2\pi)^{d/2} \mathcal{I}_{d/2-1}(\kappa)}$$

no closed-form solution

K. Mardia et al. Directional Statistics

Directional Random Variables

- periodic, nonlinear or symmetric topological structure
- uncertainty quantification
  - conventional scheme → Gaussian model in locally linearized space
  - directional statistics → parametric models inherently defined on directional manifolds



$\mathbf{Z}$ = -diag(4,3,0)

Bingham distribution

$$f_{\mathcal{B}}(\underline{x}) = \frac{1}{N(\mathbf{Z})} \exp(\underline{x}^\top \mathbf{M}\,\mathbf{Z}\,\mathbf{M}^\top \underline{x})$$

unit circle/sphere/hyperspheres $\mathbb{S}^{d-1} \subset \mathbb{R}^d$

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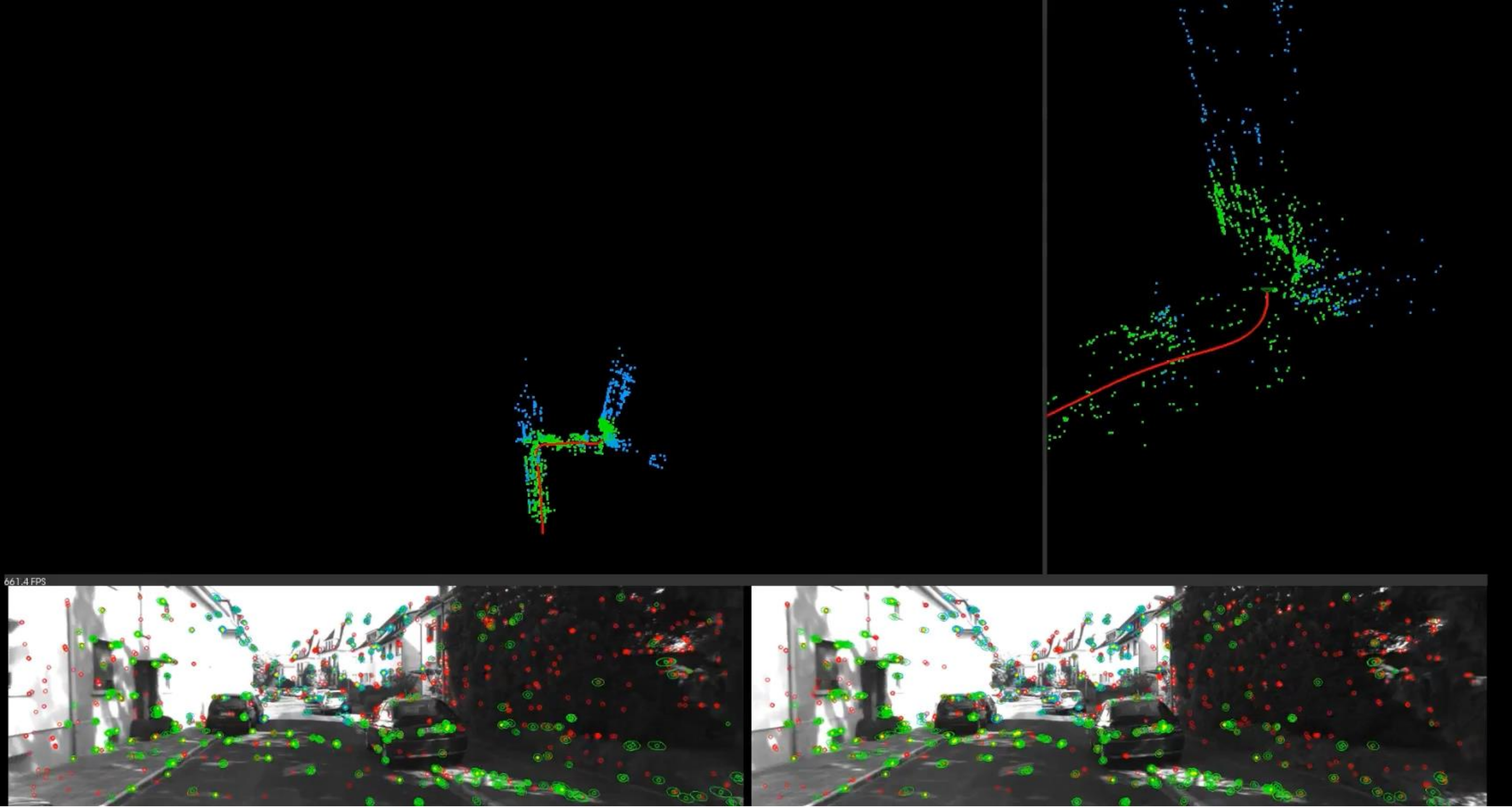
no closed-form solution

K. Mardia et al. Directional Statistics

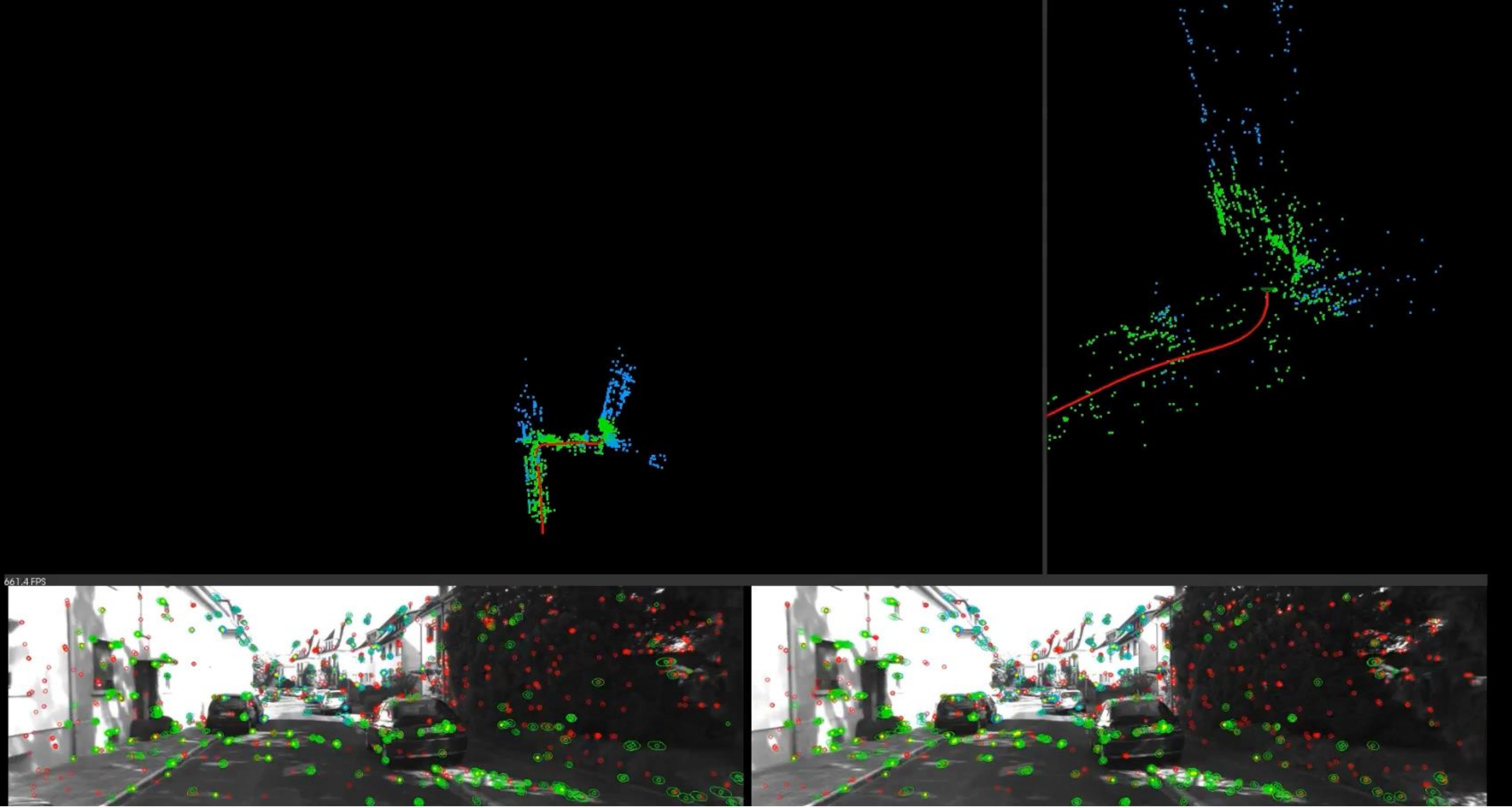


Simon Bultmann, Kailai Li, and Uwe D. Hanebeck. 22nd International Conference on Information Fusion (Fusion’19)

Direct Image Alignment

Dr.-Ing. Kailai Li

LMA-Exercise 4 | May 23, 2022



Simon Bultmann, Kailai Li, and Uwe D. Hanebeck. 22nd International Conference on Information Fusion (Fusion’19)

Direct Visual Odometry

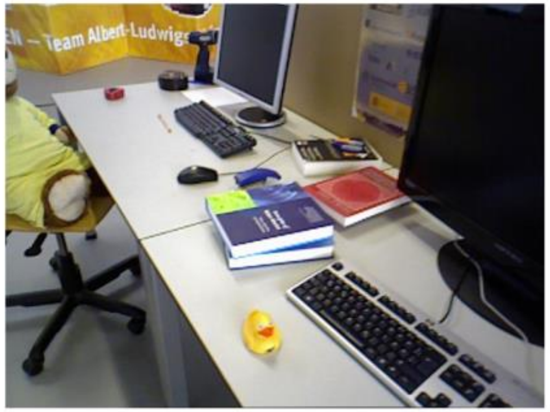
- avoid feature detection and association (manually designed)

feature-based odometry (e.g., via filtering, ICP, bundle adjustment, etc.)

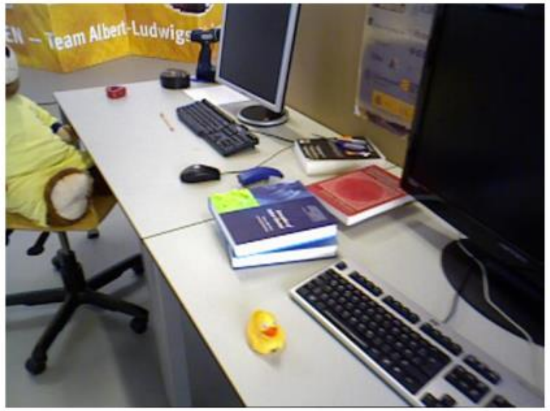
Courtesy: J. Stückler

Direct Visual Odometry

- avoid feature detection and association (manually designed)
- direct image alignment
- image warping
  - RGB-D
  - fixed-baseline stereo
  - temporal stereo, tracking and (local) mapping



I_1



I_2

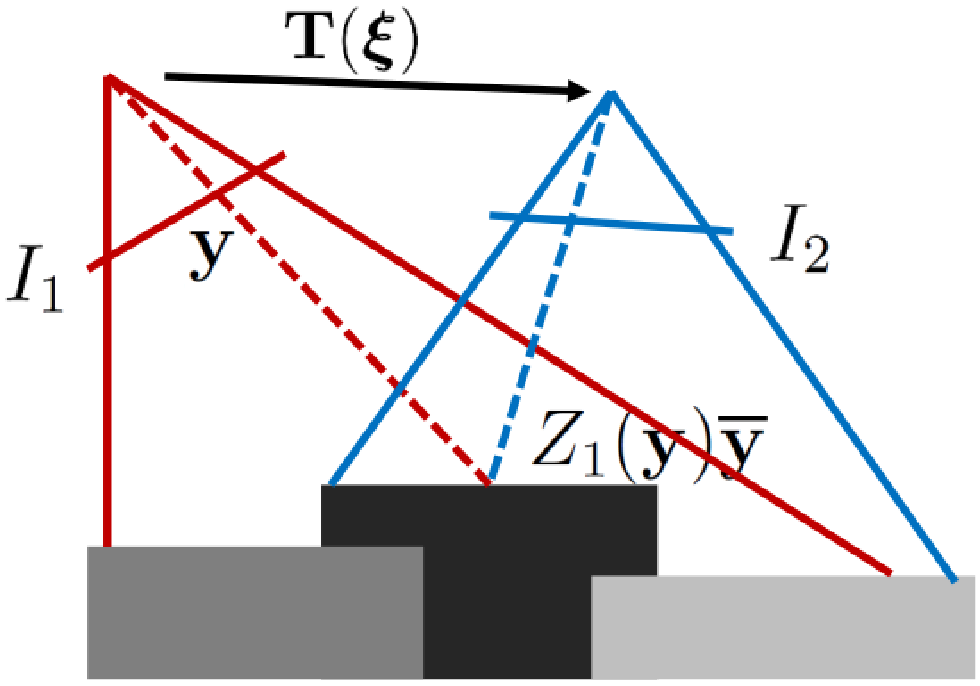
I_1 - I_2

$$\mathcal{E}(\underline{\xi}) = \sum_{u_i \in \Omega} \left\| \mathbf{I}_1(\underline{u_i}) - \mathbf{I}_2(\underline{u_i}, \underline{\xi}) \right\|^2, \quad \underline{\xi} \in \mathbb{R}^6$$

Courtesy: J. Stückler

Direct RGB-D Image Alignment

- RGB-D sensors measure intensity and depth
- warped image ideally the same as the image taken from that pose
- compute camera pose transformation by minimizing the photometric error



$$\mathbf{I}_1(y) = \mathbf{I}_2(\pi(\mathbf{T}(\xi)Z_1(y)\bar{y}))$$

Courtesy: J. Stückler

Direct RGB-D Image Alignment

- RGB-D sensors measure intensity and depth
- warped image ideally the same as image taken from that pose
- compute camera pose transformation by minimizing the photometric error
  - assumes that pixel measurements are stochastically independent
  - nonlinear least square problem
  - efficient optimizers using standard second-order tools (Gauss-Newton, LM) available

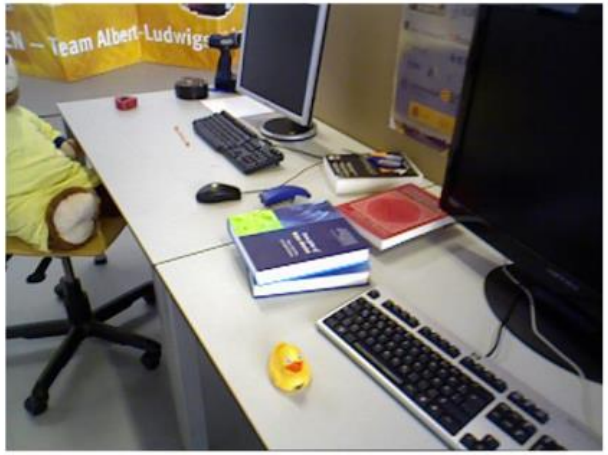
$$\mathbf{I}_1(y) = \mathbf{I}_2(\pi(\mathbf{T}(\xi) Z_1(y)\bar{y})) + \epsilon\,, \quad \epsilon \sim \mathcal{N}(0, \sigma^2)$$

$$\xi^* = \arg\min_{\xi} \sum_{y_i \in \Omega} r(y_i, \xi)^2 / \sigma_I^2$$

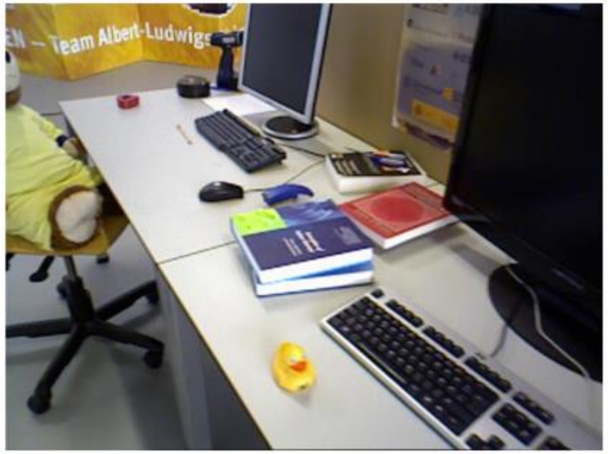
$$\text{residuals: } r(y_i, \xi) = \mathbf{I}_1(y_i) - \mathbf{I}_2(\pi(\mathbf{T}(\xi) Z_1(y_i)\bar{y_i}))$$

Courtesy: J. Stückler

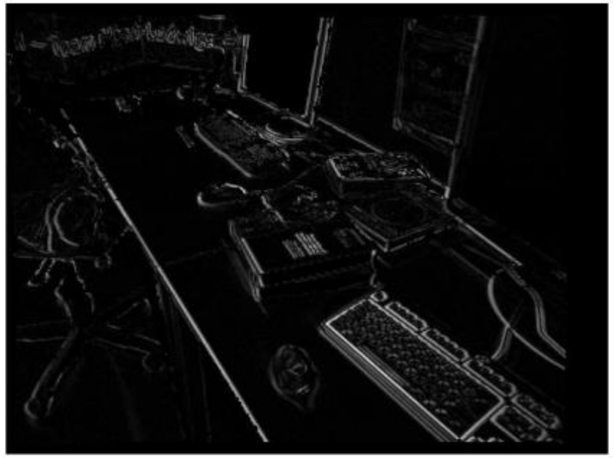
Direct RGB-D Image Alignment



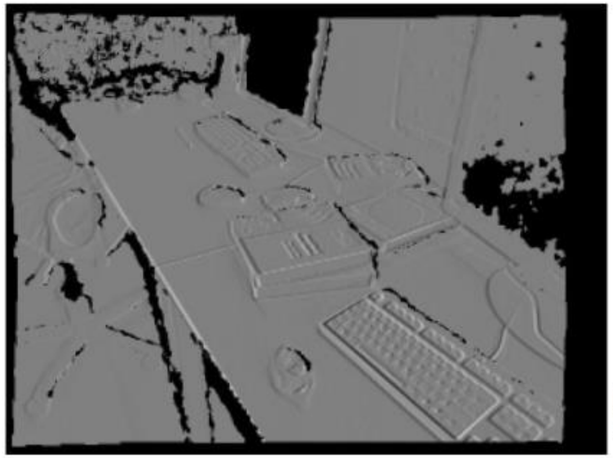
I_1



I_2



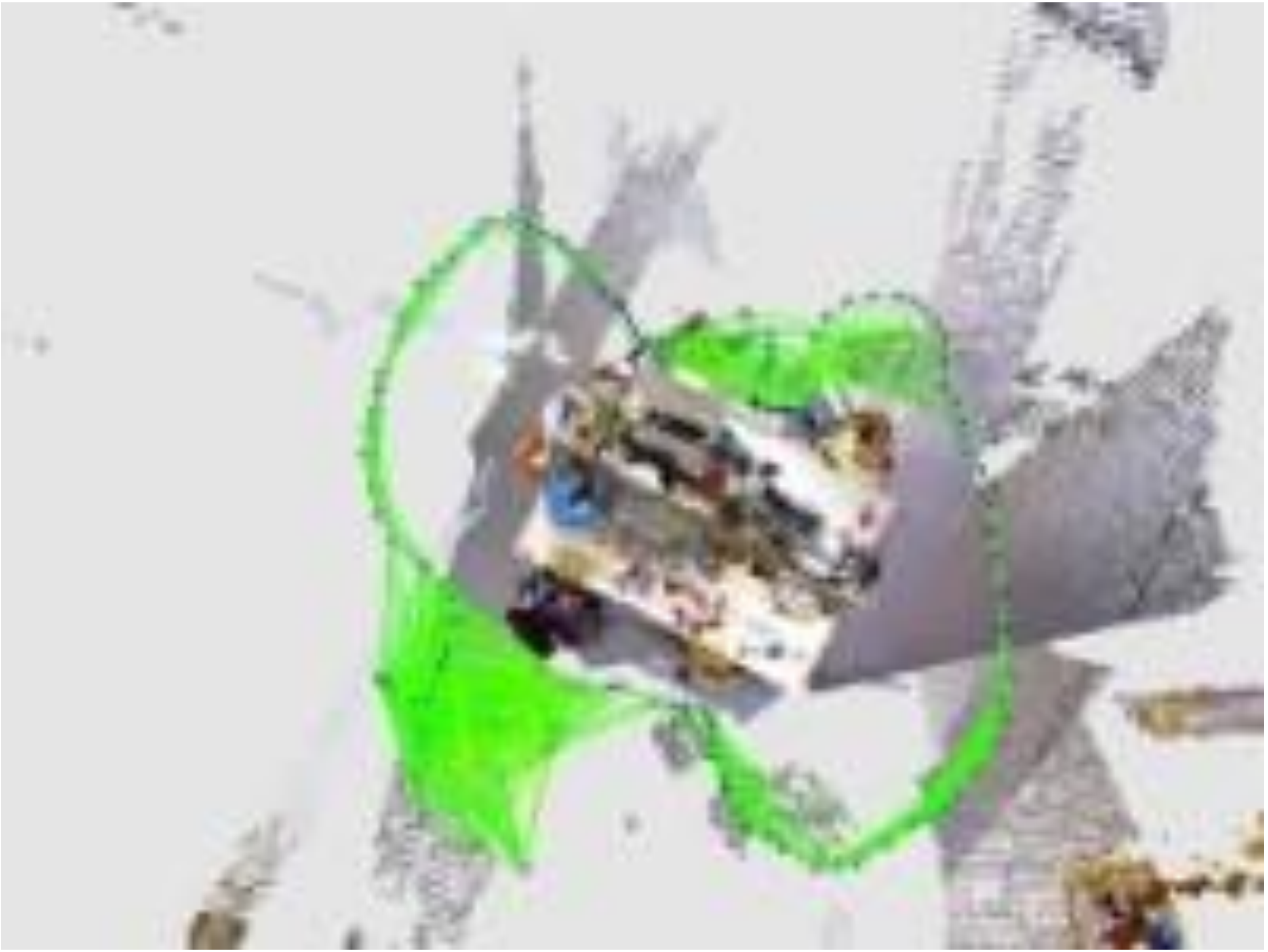
I_1 - I_2



$$\left.\frac{\partial I_2\left(\pi\left(\mathbf{T}(\boldsymbol{\xi}) Z_1(\mathbf{y})\overline{\mathbf{y}}\right)\right)}{\partial v_x}\right|_{\boldsymbol{\xi}=\mathbf{0}}$$

Courtesy: J. Stückler

Direct RGB-D Image Alignment



Map Representations

Dr.-Ing. Kailai Li

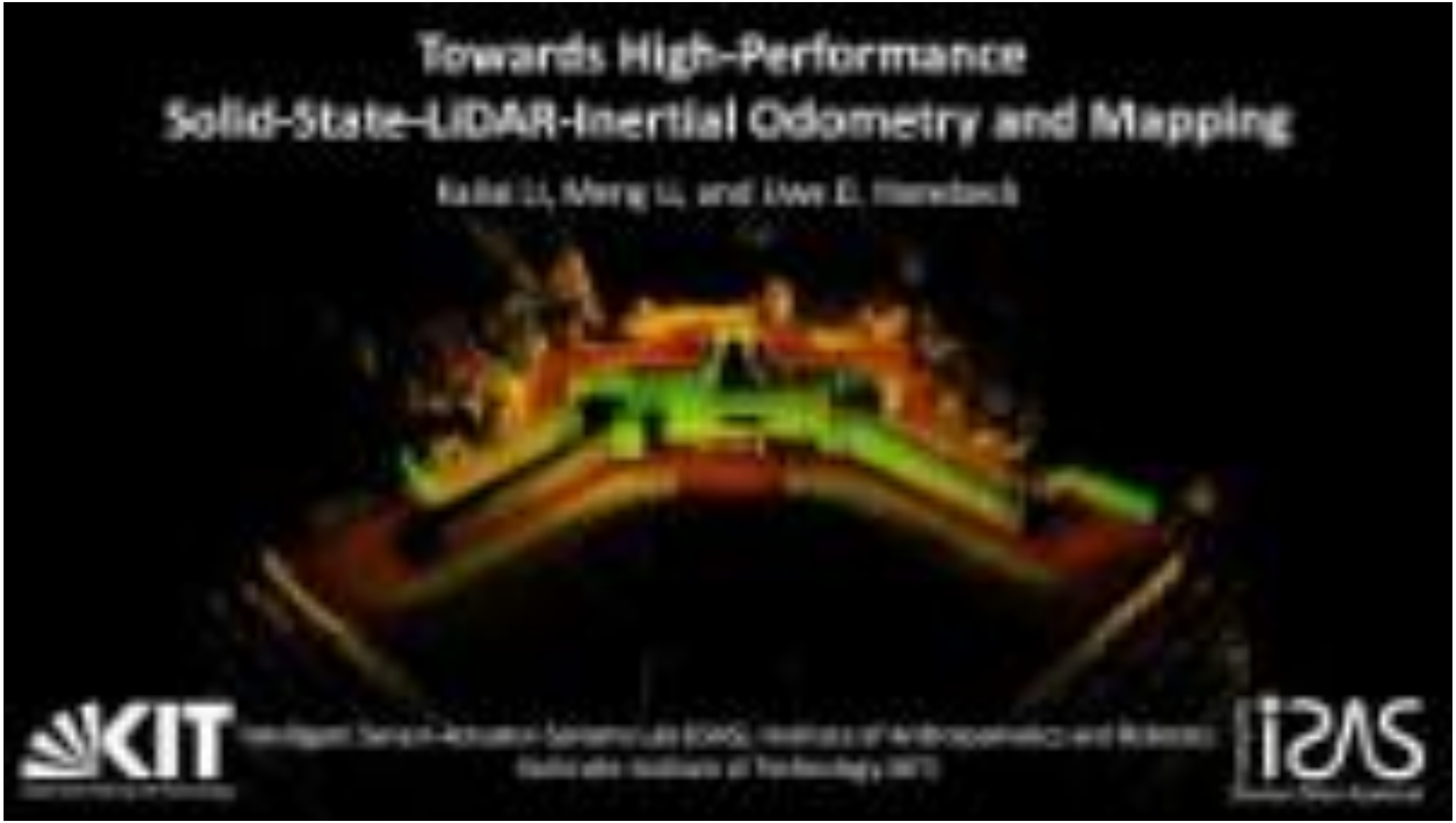
LMA-Exercise 5 | May 30, 2022

Mapping Approaches

- sparse vs. dense
- probabilistic vs. deterministic
- explicit vs. implicit
- raw vs. geometric primitives
- examples:
  - point clouds
  - occupancy grids
  - surfels
  - signed distance function (SDF) -> truncated signed distance function (TSDF)

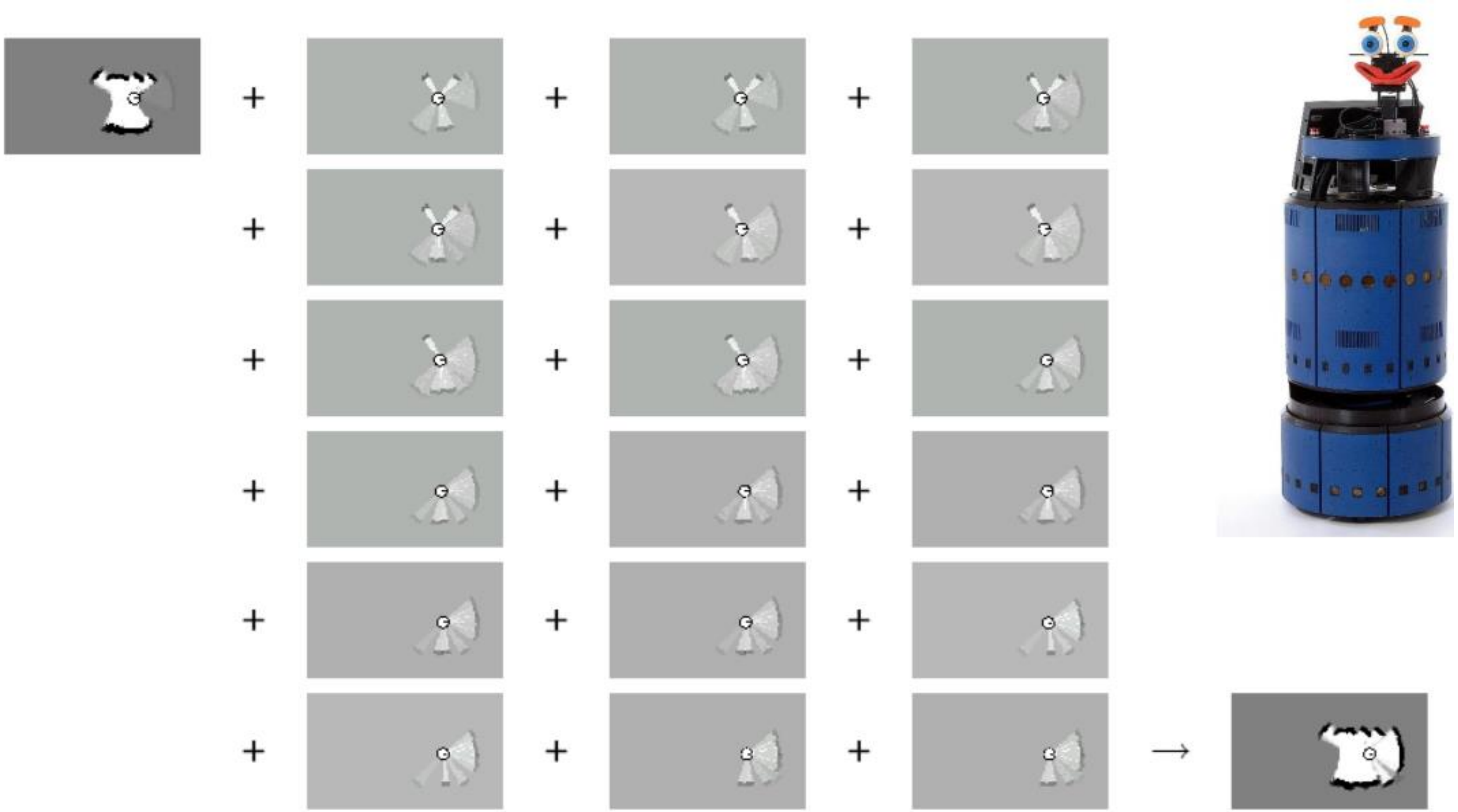
Courtesy: J. Stückler

Point Clouds



3D, dense/sparse, deterministic, explicit, (quasi-)raw mapping approach

Occupancy Grid Map

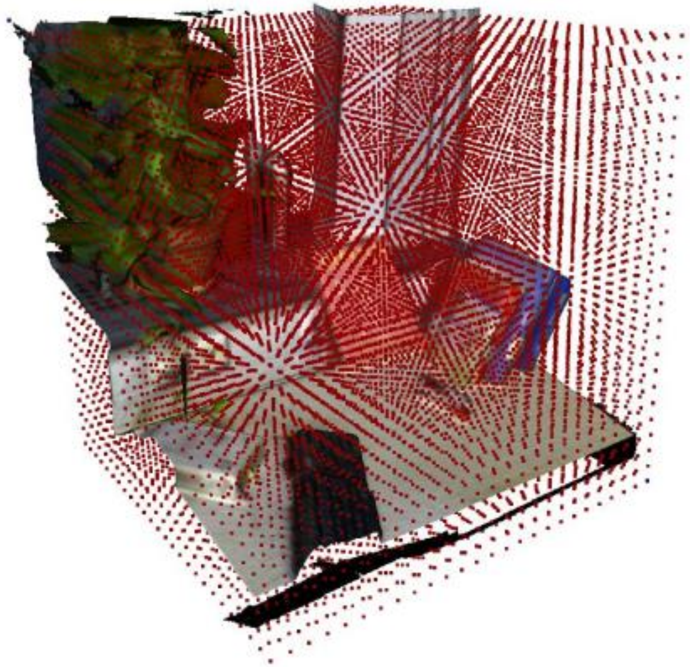


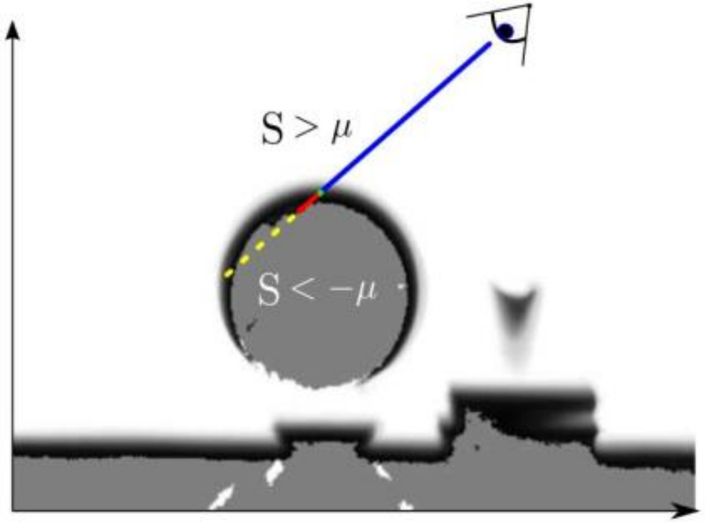
2D/3D, explicit, probabilistic mapping approach

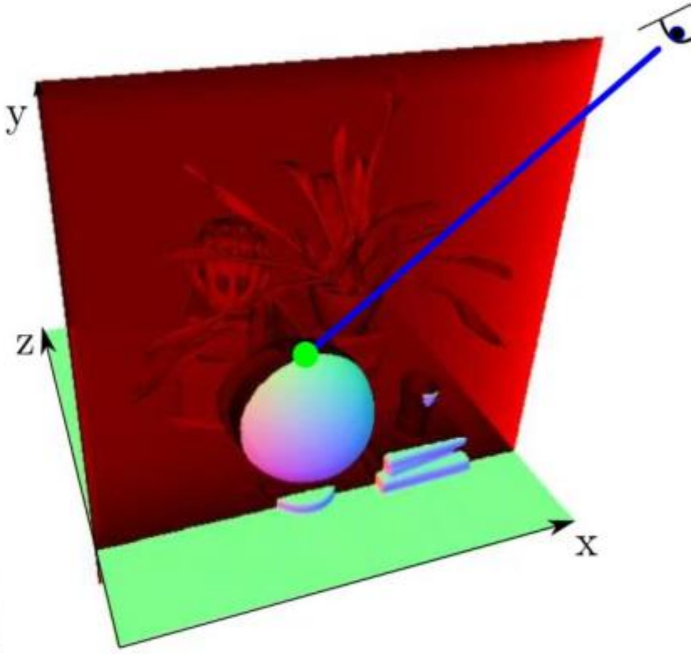
Courtesy: S. Thrun et al. 2015.

Truncated Signed Distance Function (TSDF)

- 3D volumetric map for 40m x 40m x 40m with 0.05m resolution
  - ➢ 40^3/0.05^3 = 512,000,000 voxels (4.096 GB at double precision)
- However, large amount of volumes are actually empty.
- 3D, implicit, deterministic, dense (typically), mapping approach







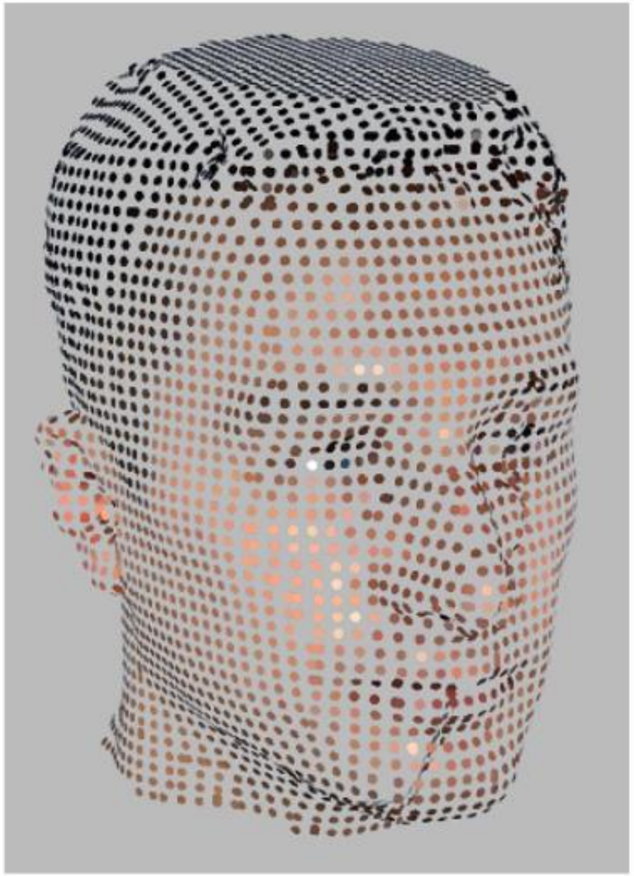
Courtesy: Bylow et al., 2013; Newcombe et al., 2011.

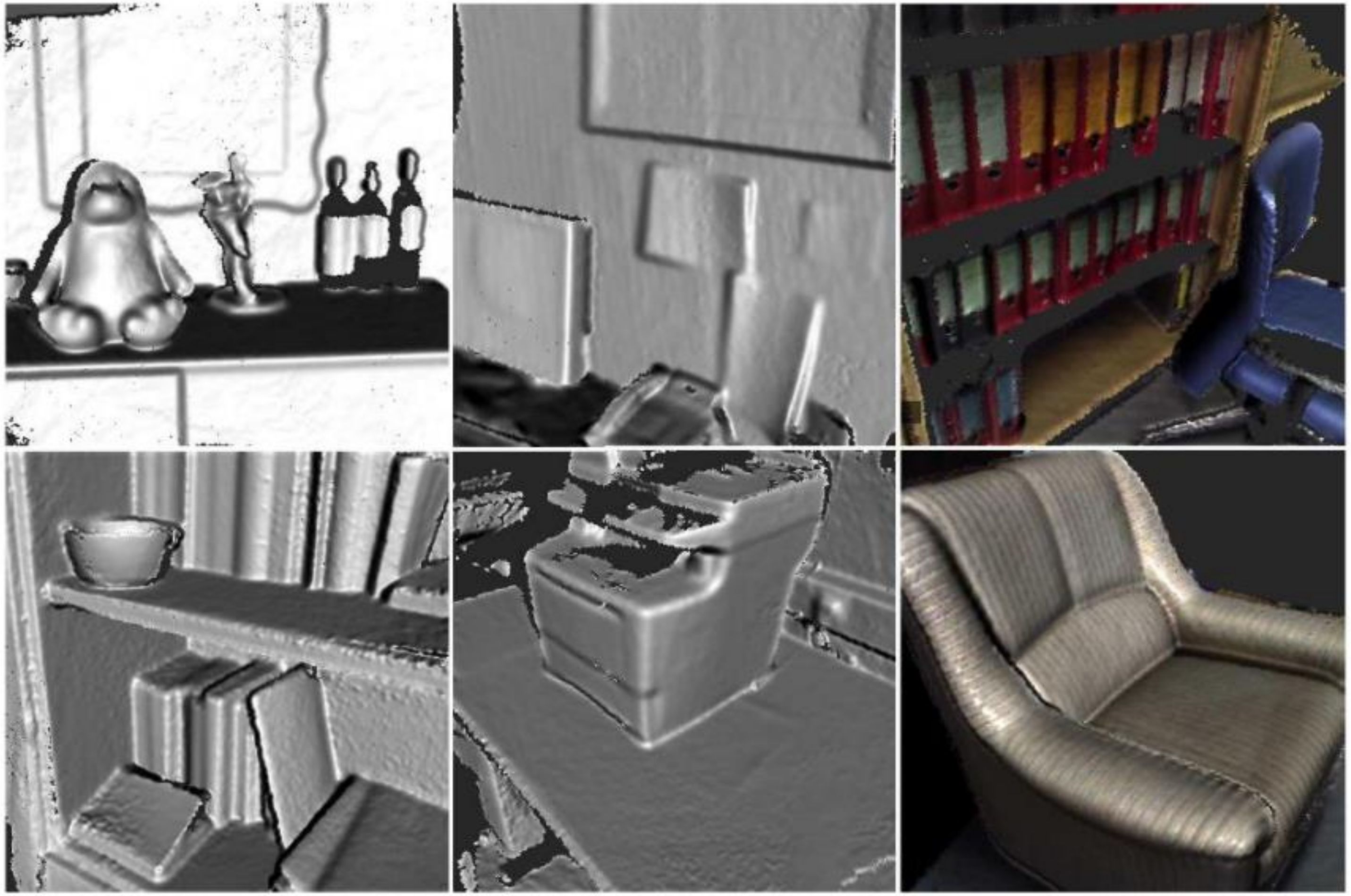
Truncated Signed Distance Function (TSDF)

Truncated Signed Distance Function (TSDF)

Surfels

- surfel: point + normal + radius



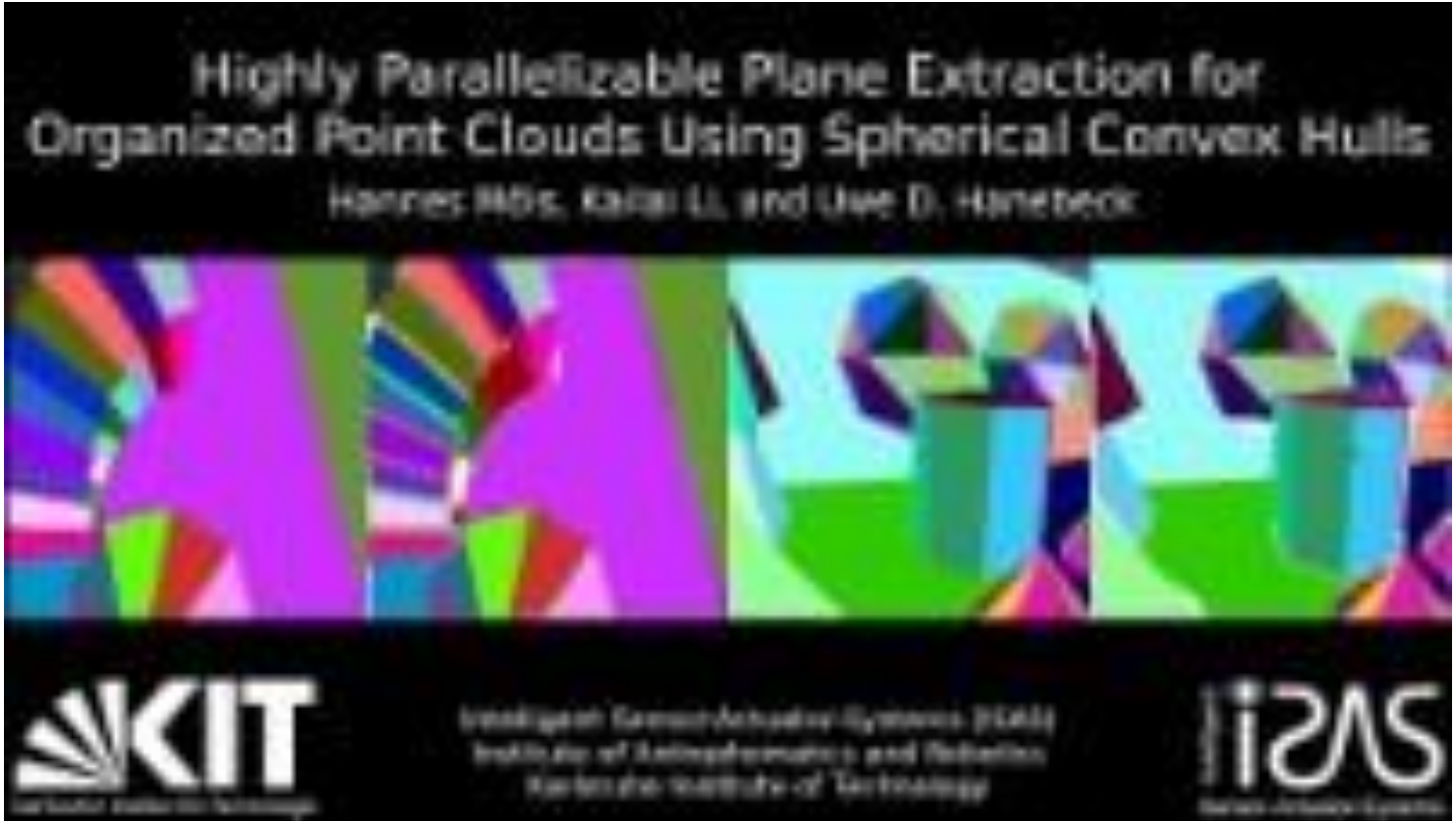


3D, deterministic, explicit, dense (typically) mapping approach

Courtesy: M. Zwicker, Keller et al., 2013.

Surfels

Geometric Primitives



2D/3D, deterministic, explicit, dense/sparse mapping approach

Graph-Based SLAM

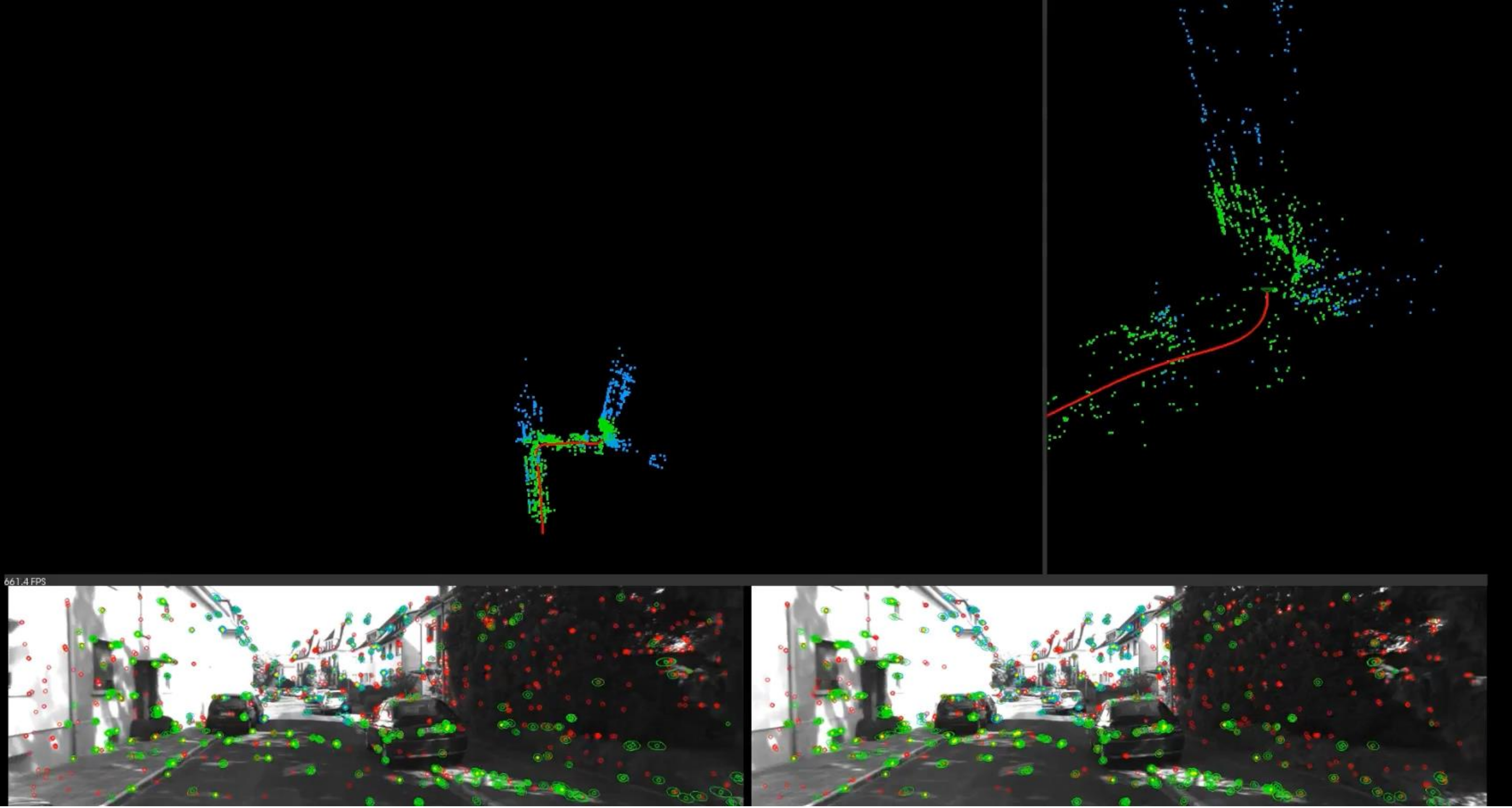
Dr.-Ing. Kailai Li

LMA-Exercise 9 | July 11, 2022

Background

- Joint pose and map estimation
- Chicken-egg problem
- Fundamental task for autonomous robots
- Basis for navigation systems

Courtesy: J. Stückler



Simon Bultmann, Kailai Li, and Uwe D. Hanebeck. 22nd International Conference on Information Fusion (Fusion’19)

Formulation

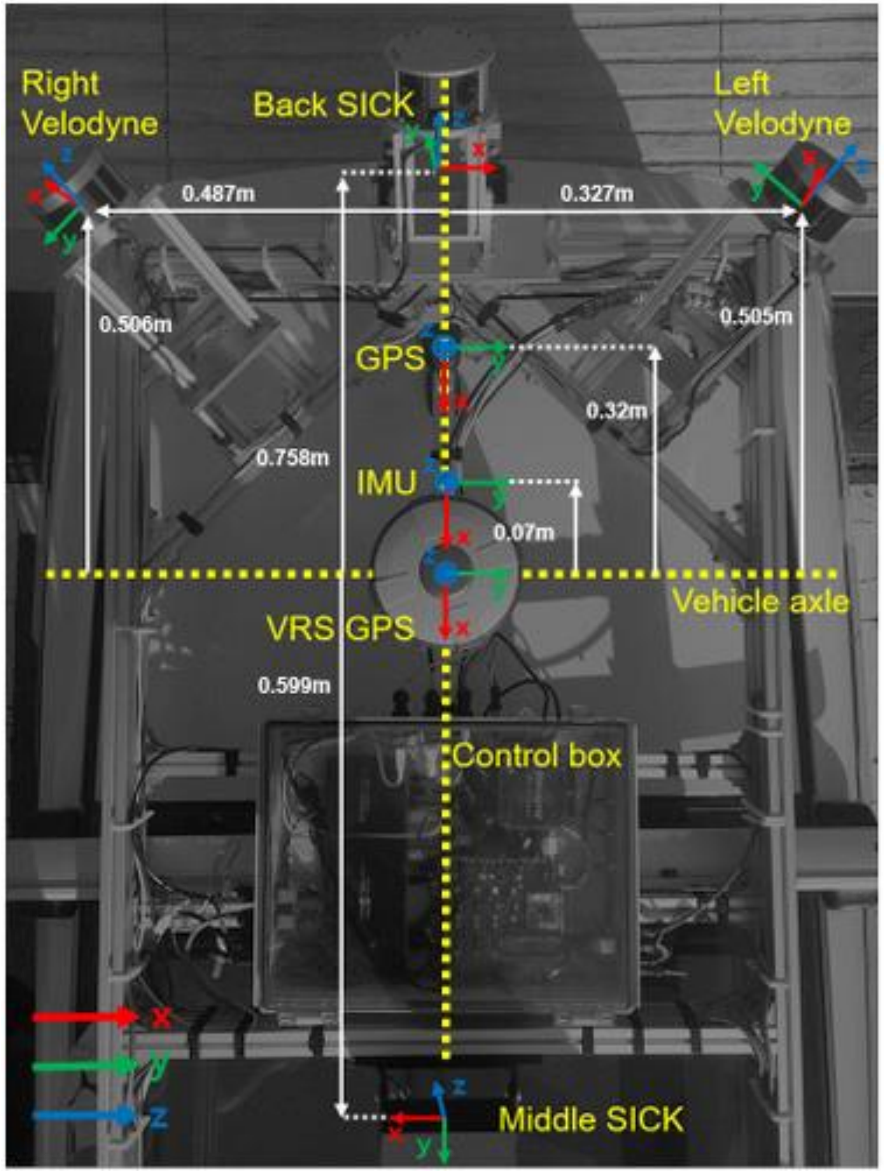
$$p(x_{1:\tau}, \mathcal{M} \mid z_{1:\tau}, u_{1:\tau}, x_0)$$

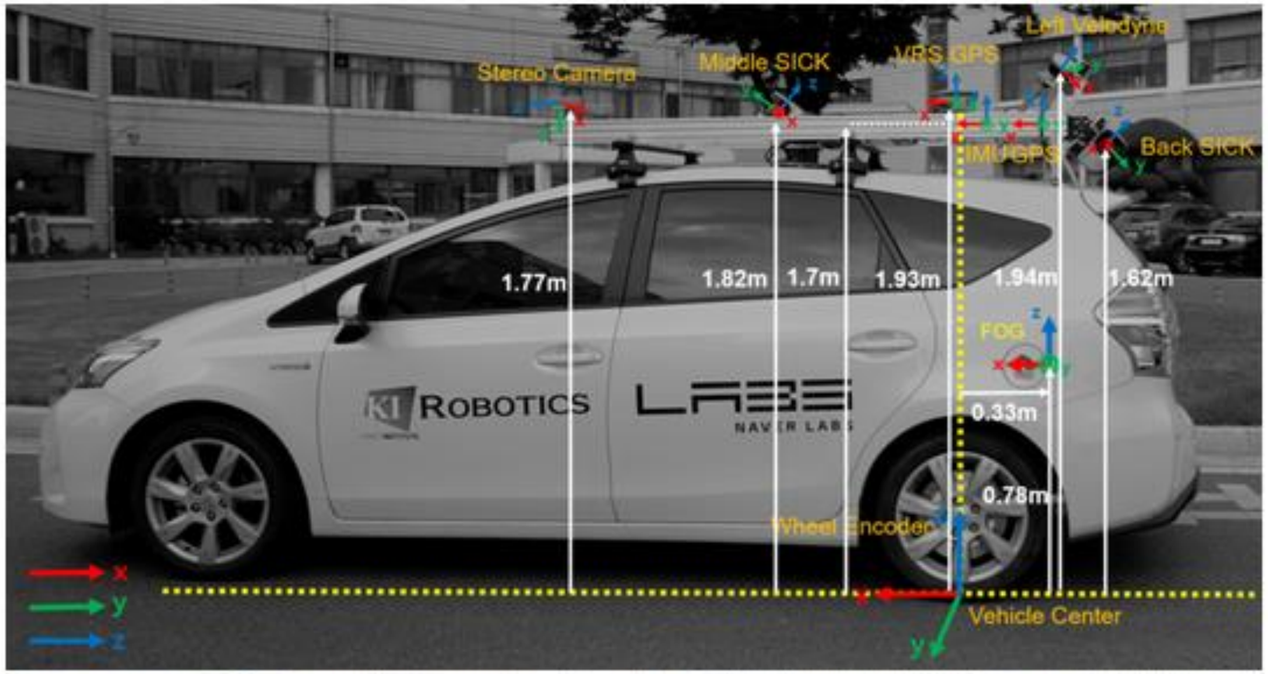
state map observation input initial state

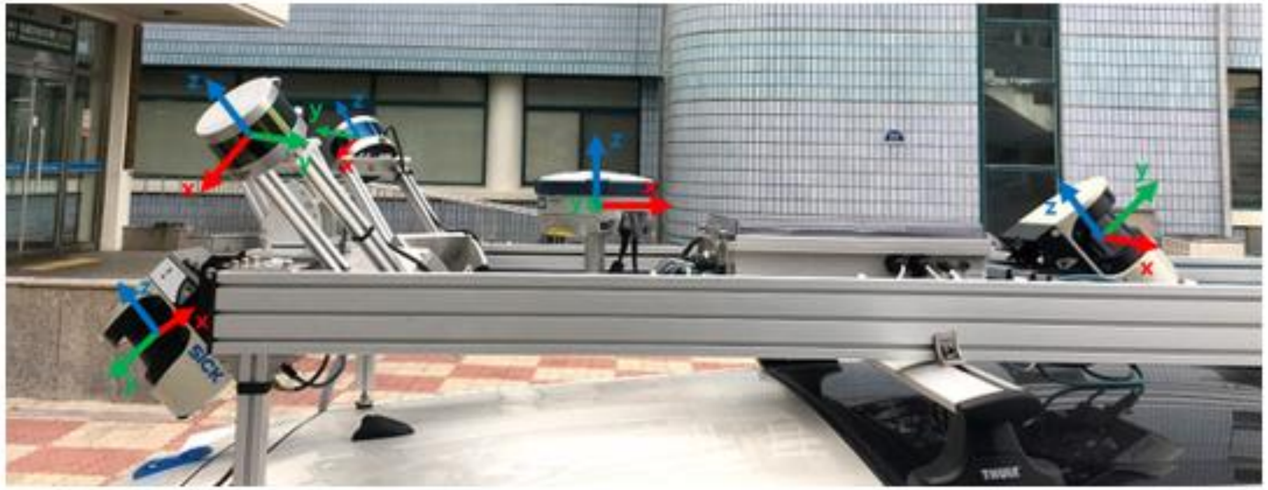
- State: SE(2)/SE(3) pose, speed, IMU biases, etc.
- Map: various representation approaches w.r.t. effeciency, accuary, semantics, etc.
- Observation: various sensory modalities
  - Visual sensors: monocular/stereo camera, RGB-D cameras
  - Range sensors: ultrasonic, radar, UWB, LiDAR (2D/3D)
  - Others: GNSS, wifi, etc.
- Input: given as orders, or measured by onboard sensors
  - Wheel odometry
  - Inertial measurement unit (IMU)

Courtesy: J. Stückler

Sensors







Courtesy: Jinyong Jeong et al.

System Pipeline Design

Karlsruhe Institute of Technology

Frontend

- Convert raw sensor data into an immediate and intermediate representation
  - Constraints for optimization, e.g., feature correspondences
  - Probability distributions of landmarks
  - Relative trasnformatoin between frames
  - Etc.
- Very task-specific: recursive estimators, iterative closest point, direct image alignment

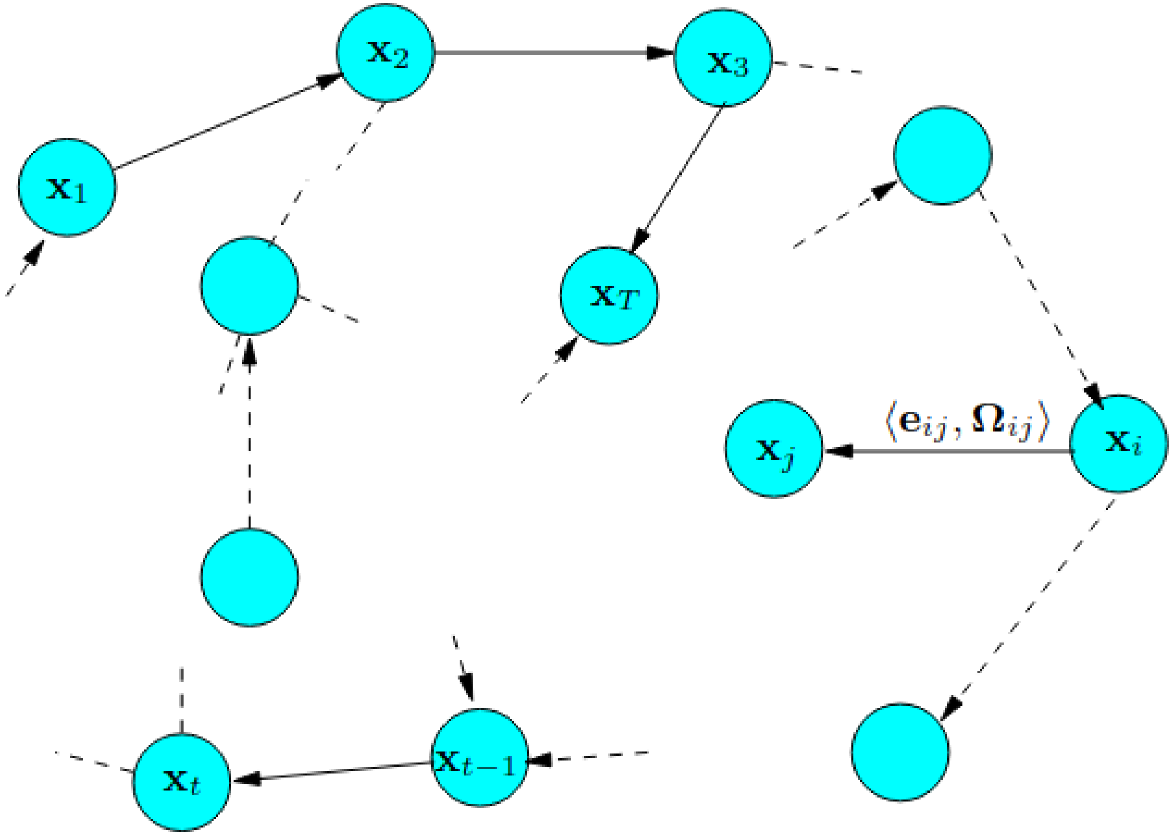
Backend

- Takes intermediate representatoin and solves the underlying state estimation or optmization problem
- Category of approaches
  - Extended Kalman filter (EKF) -> EKF-SLAM
  - Particle filter (PF) -> FastSLAM
  - Nonlinear least squares -> graph SLAM

Courtesy: J. Stückler

Graph-Based SLAM

- Uses a graph to represent the problem, namely, the variables (nodes) and the relations between variables (edges)
- edges placed between variable nodes representing prior information or information from the frontend

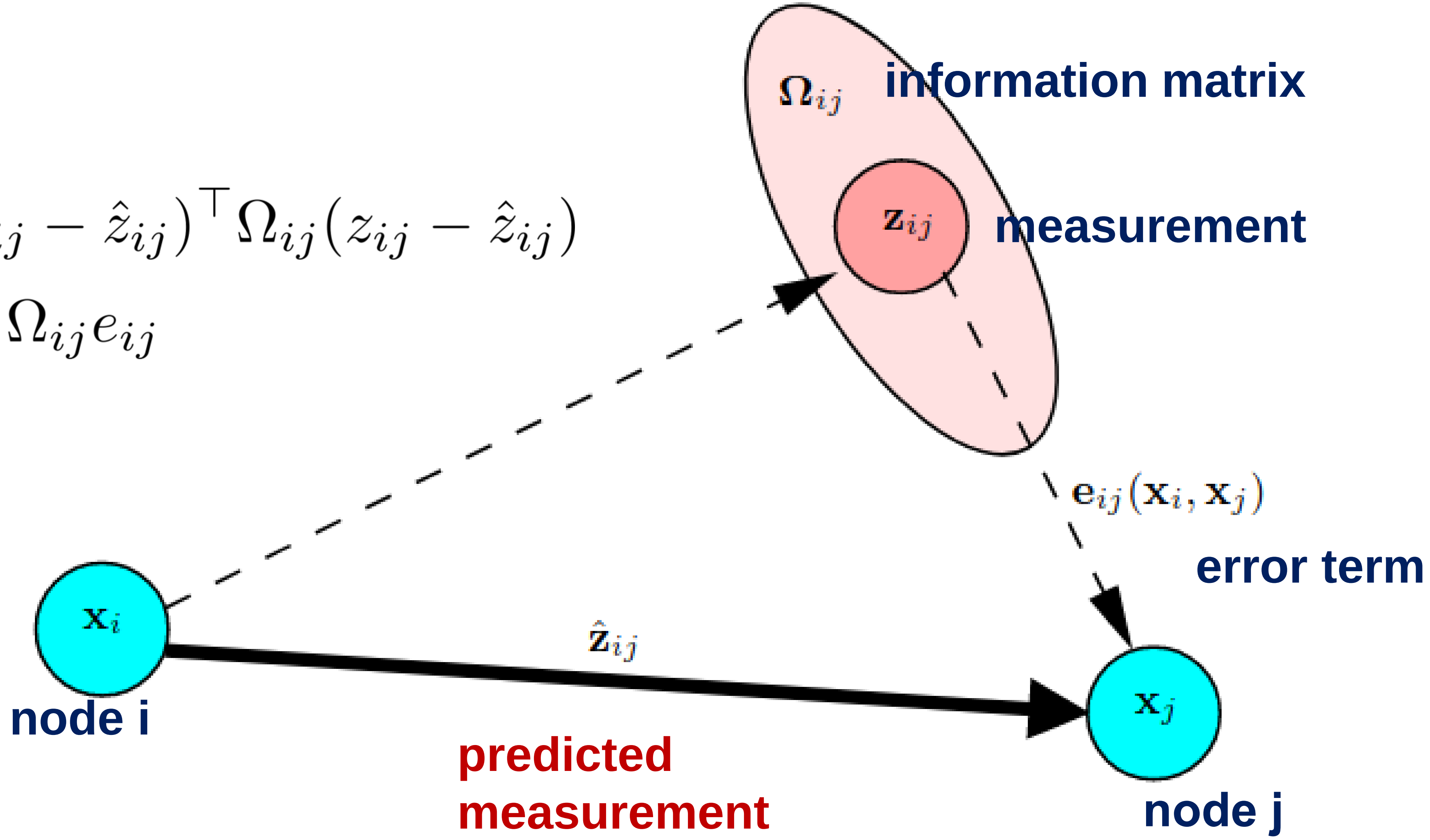


Courtesy: G. Grisetti et al.

Formulation

KIT
Karlsruhe Institute of Technology

$$l_{ij} \propto (z_{ij} - \hat{z}_{ij})^\top \Omega_{ij} (z_{ij} - \hat{z}_{ij})$$
$$= e_{ij}^\top \Omega_{ij} e_{ij}$$



Courtesy: G. Grisetti et al.

Formulation

$$x^* = \arg\min_x \sum_{<i,j>\in\mathcal{C}} e_{ij}^\top \Omega_{ij} e_{ij} \text{ , with } x = [x_1^\top, ..., x_\tau^\top]^\top$$

- Implemented through adjacency matrix of the graph
- Nonlinear least square problem
  - Linearize the error function (Taylor expansion)
  - Compute its derivative
  - Set it to zero
  - Solve the linear system
  - Iterate until convergence

Courtesy: G. Grisetti et al.

Linearization

$$e_{ij}(x+\Delta x) = e_{ij}(x) + \mathbf{J}_{ij}\Delta x \,, \text{with}\, \mathbf{J}_{ij} = \frac{\partial e_{ij}(x)}{\partial x}$$

- Error function for one edge only depends on the two states on the two nodes
- Jacobian
  - non-zero only in the rows corresponding to the two nodes
  - Sparse structure

$$\mathbf{J}_{ij} = \frac{\partial e_{ij}(x)}{\partial x} = \left(0 \ldots \frac{\partial e_{ij}(x)}{\partial x_i} \ldots \frac{\partial e_{ij}(x)}{\partial x_j} \ldots 0\right)$$

$$= \left(0 \ldots \mathbf{A}_{ij} \ldots \mathbf{B}_{ij} \ldots 0\right)$$

Courtesy: G. Grisetti et al.

Linearization

$$\mathbf{F}_{ij}(\breve{x} + \Delta x) = (e_{ij} + \mathbf{J}_{ij}\Delta x)^\top \Omega_{ij} (e_{ij} + \mathbf{J}_{ij}\Delta x)$$
$$= e_{ij}^\top \Omega_{ij} e_{ij} + 2 e_{ij}^\top \mathbf{J}_{ij} e_{ij} \Delta x + \Delta x^\top \mathbf{J}_{ij}^\top \Omega_{ij} \mathbf{J}_{ij} \Delta x$$
$$=: c_{ij} + 2 b_{ij} \Delta x + \Delta x^\top \mathbf{H}_{ij} \Delta x$$

$$\mathbf{F}(\breve{x} + \Delta x) = \sum_{<i,j> \in \mathcal{C}} c_{ij} + 2 b_{ij} \Delta x + \Delta x^\top \mathbf{H}_{ij} \Delta x$$
$$=: c + 2 b^\top \Delta x + \Delta x^\top \mathbf{H} \Delta x$$

- First-order approximation of residual over two nodes
- Sparse structure for Hessians
- Zero first derivative leads to solving linear system $\mathbf{H}\Delta x^* = -b$

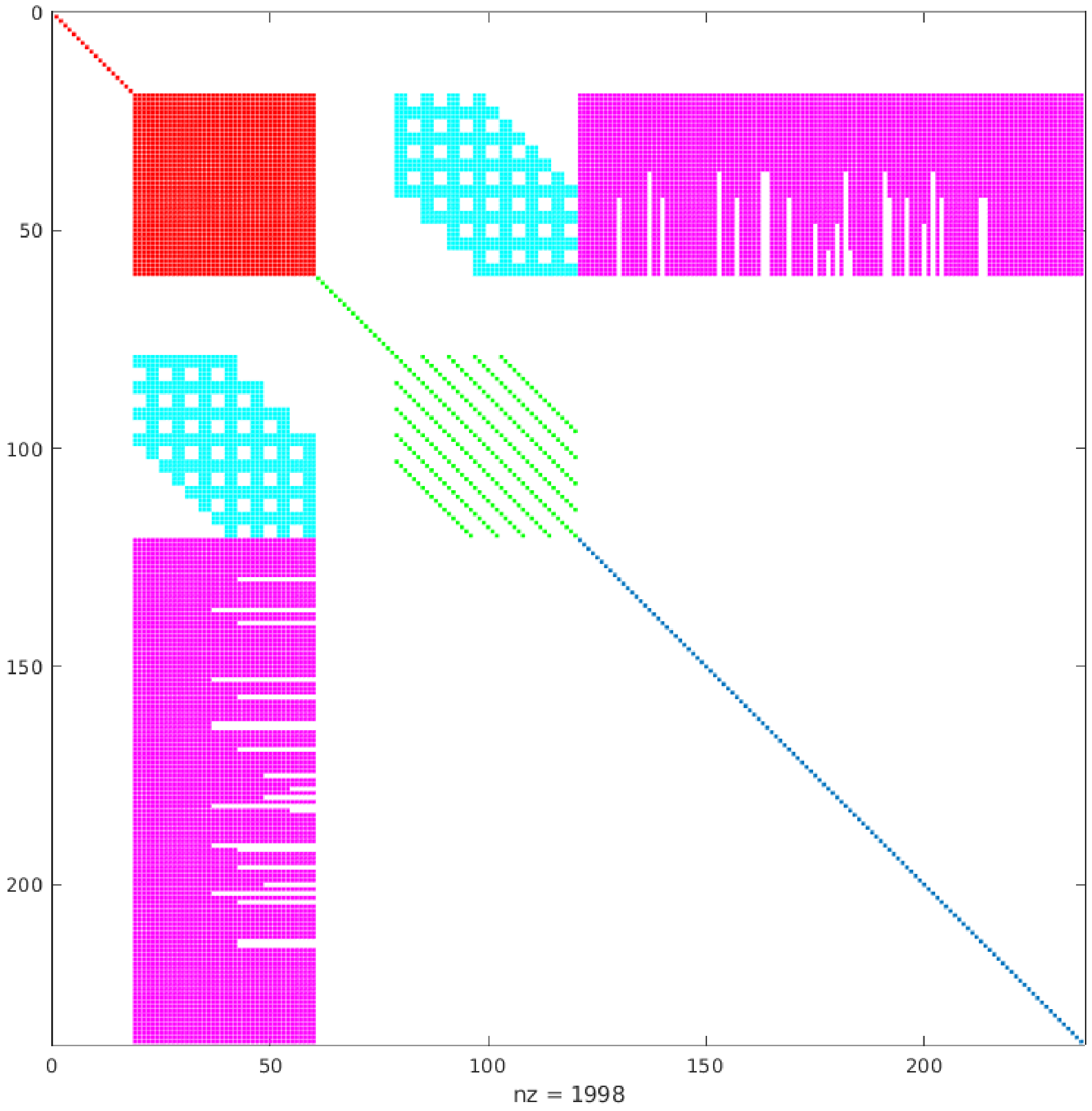
Courtesy: G. Grisetti et al.

Linearization

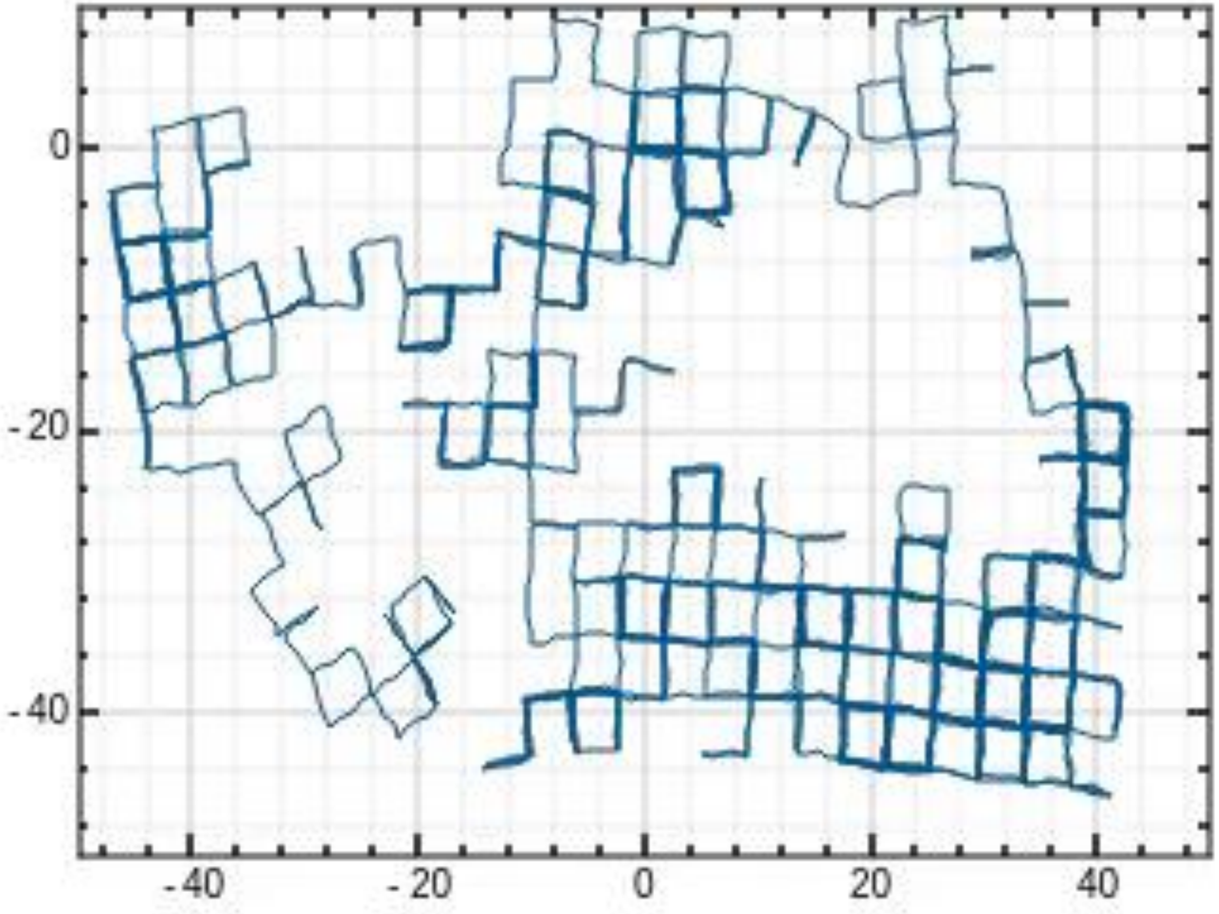
pose

bias

inverse
depths



Optimization



Kailai Li, Johannes Cox, Benjamin Noack, and Uwe D. Hanebeck. IFAC ’20

Sequence FR-IOSB-Long

8x

LiLi-OM-ROT (HDL-64E)

LiLi-OM (Horizon)

Graph-Based SLAM

Summary

- Very useful and universally applicable in most mobile robotic applications (as backend)
- Mathematical formulation also used for frontend
- Softwares:
  - g2o
  - gtsam
  - ceres

Courtesy: C. Stachniss *et al.*